Conflicting treatment model for certainty rule-based knowledge

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Abstract

The rule-based knowledge based expert system has traditionally emphasized the verification of structural errors in the rule base. For conflicting or overlapping rules, designated rules are usually followed to implement prioritized or direct deletions. However, there exist no proper methods by which to resolve conflicts, inconsistencies or redundancies in values. The citation of erroneous knowledge can lead to mistakes in reaching decisions.

This study proposes the conditional probability knowledge similarity algorithm and calculation system. The calculation system can quickly and accurately calculate rule-based knowledge similarity matrices and determine the conflicting or overlapping rules. Employing the group decision idea, an algorithm is provided that uses a “reliability factor” to refer to the reliability level of the knowledge item with a conflict, redundancy or inconsistency in value, and constructs a conflict treatment model for certainty rule-based knowledge.

Most users, 94% report perplexity at the moment that conflicting or redundant rules are cited. Moreover, 92% of users hold that the algorithm is helpful to knowledge application and as an aid to the decision-making process. It can more effectively prevent mistakes in decision making and enable users to acquire more benefits from the knowledge application.

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1. Introduction

A rule-based knowledge base expert system is a rule base formed after the combination of facts obtained by the experts familiar with all realms and rules. However, there is always a possibility of conflicts in the experience of experts engaged in diversified fields. Hence a rule-based knowledge base expert system tends to value verification of the rule base. More effort is devoted to checking for structural errors in order to ensure the quality of the knowledge base and correctness in inference.

Verification of the rule base concentrates on detecting structural errors resulting from interaction between rules. Structural errors may influence the consistency of rule inference through redundancy, inconsistency, incompleteness and circularity (Geissman & Schultz, 1988; Naugan, Perkins, Alford, & Pecora, 1987; Nazareth, 1989; Siuwa, Scott, & Shortliffe, 1982; Tsai, Vishnuvajula, & Zhang, 1999). To define terms, structural errors include redundancy, inconsistency, incompleteness and circularity of the rule. Incompleteness refers to a lack of completeness when the general rule is used to express the designated field, which covers unnecessary conditions, dead-end rules or dangling conditions. Structural errors will directly or indirectly influence the inference drawn from the system rule. Structural errors are distinguished as direct or indirect according to whether they exert an immediate influence over the rule system. Inconsistent rules are direct structural errors while redundant rules, conflicting rules and circular rules are indirect structural errors.

Referring to Table 1, the currently prevailing method of treating redundancy and conflict between rules is to implement prioritized or direct deletion by following designated rules. However, there exist no proper methods to resolve...
conflicts, inconsistencies or redundancies in value. Redundancy and inconsistency are likely to result in the incomplete fulfillment of the benefits intended to be derived from the knowledge application. Worse, they may produce a negative effect. Additionally, the uncertainty in the uncertain knowledge itself adds difficulty to the treatment of conflicts between uncertain knowledge of various forms. On the other hand, citation of erroneous knowledge will lead to mistakes in making a final decision. To obtain more benefits from knowledge management applications, it is a necessity to confirm the existence and reliability of cited knowledge.

Employing a group decision rule, the current study attempts to provide a certainty rule-based knowledge conflict treatment algorithm for certainty rule-based knowledge. In the algorithm, the term “reliability factor” refers to the reliability of the knowledge in which there is a conflict, overlap or inconsistency in value. For conflicting or redundant knowledge, knowledge of higher reliability can be chosen and extensively applied to treatment of all certainty rule-based conflicting or redundant knowledge. In this way, mistaken decisions can be effectively prevented and more benefits acquired from the knowledge application.

2. Literature review

2.1. Knowledge representation

As a rule, knowledge can be represented in the following ways: as Rule-based Knowledge, Frame-based Knowledge, a Semantic Network, Case-based Knowledge and Ontology.

Rule-based Knowledge is the most popular and common way to represent knowledge (Zadeh, 1992; Negnevitsky, 2002). In this representation, the knowledge is represented in a set or sets of rules. The representation model is IF (pre-requisite) plus THEN (conclusion). Frame-based knowledge was proposed by Minsky in 1975. It is formed mainly by the aggregate of knowledge slots. Each slot is the aggregate of knowledge facets and each facet has its corresponding slot value. Each frame is used to represent the relevant knowledge concept or knowledge object (Minsky, 1975). The structure for frame-based knowledge is shown as follows:

```lisp
  ((frame name))
  ((slot1)(facet1)(value1))
  ((slot2)(facet2)(value2))
```

The semantic network is represented by the directed graph where the node expresses the component of the knowledge or type in the knowledge of a specific field; the arc between nodes indicates the relation between components of the knowledge (Quillian, 1968). Case-based knowledge is usually used to describe knowledge as part of an experience. Ontology expresses the meaning of the existence of knowledge or the being of knowledge (Vet & Mars, 1998).

2.2. Knowledge similarity

According to statistics provided by McGill, Koll and Noreault in 1979, current methods for measuring similarity were continually growing and already numbered more than 60 types, including inner product, Dice coefficient, cosine coefficient, Jaccard coefficient, overlap coefficient, etc. (Zhang & Rasmussen, 2001). However, the most popular method today remains one based on the distance between the two end points of two vectors and the angle between the two vectors. Generally, the two-vector similarity measure is represented by the distance between the two end-points of the two vectors or the angle between the two vectors. The less the distance or the angle is, the higher the similarity. Distance in a geometric distance model is usually represented by Euclidean distance; and the angle by dot product (Falkowski, 1998).

Chen et al. observed that comparison of the similarity measures of two fuzzy values mainly adopts the geometric distance model, the union and intersection operations in the set-theoretic approach, and the sum and difference of grades of membership and matching functions (Chen, 2000; Chen, 2004; Chen, Yeh, & Hsiao, 1995). In 1999, Zhang put forward the similarity measure method integrating distance and angle. In this method, distance similarity uses an exponential function, with the bottom between 0.7 and 0.97, and angle similarity a cosine function (Zhang & Rasmussen, 2001; Zhang & Korfhage, 1999). In 2005, Liu proposed the use of a sequence alignment and description, together with the Lempel-Ziv Algorithm, to calculate the similarity of DNA gene sequences (Liu & Wang, 2005). In 2004, Yang et al. proposed the optimal classification method which mainly uses an exponential function as the similarity goal function (Yang & Wu, 2004).

2.3. Redundant rules and conflicting rules

On the condition that a group of the same antecedents are preset, if the same conclusion is inferred from two rules or the aggregate of rules from different groups, then the existence of redundant rules in the rule base is indicated. The reason for the existence of the redundant rules is likely due to inclusive relations between antecedents or consequents and the condition that on the basis of a group of antecedents, many rules come to the same middle point or consequent when an inference is made through different inference paths.
Take the following three rules, for example. R3 can be identified as a redundant rule according to R1 and R2.

R1: IF a1 THEN c1
R2: IF c1 THEN c2
R3: IF a1 THEN c3

If the antecedents remain the same, complete oppositeness in consequents or inconsistency in value may speak of conflicting rules. Conflicting rules may lead the expert system or knowledge user to mistaken decisions. For instance, it can be seen from the following three rules that R4 and R5 are mutually conflicting because R4 proves to be a conflicting rule according to the inference from R5 and R6.

R4: IF a1 THEN c4
R5: IF a1 THEN c7
R6: IF c2 THEN Not c1

2.4 Methods of treating knowledge conflicts or overlaps

Suwa et al. put forward a set of verification checkers to detect conflicting, overlapping and inclusive rules (Yang & Wu, 2004). Cugun et al. proposed a decision table base processor to cut and rank the main table into several sub-tables to check whether errors exist (Cugun & Steudel, 1987). The high-order Petri Nets, together with graph and mathematical theory, can be used to represent the system (Zhang & Nguyen, 1994; Murata, 1988) by observing the sensor and forecasting the transfer network model. Although conflicting rules and overlapping rules can be detected, the overlapping rules are usually directly deleted. As for the conflicting rules, they will be often treated in chronological order by the designated rules. However, there is no appropriate method of treating the conflict between rules, inconsistency in corresponding value or overlapping between rules. The present study attempts to provide a certainty rule-based knowledge conflict treatment algorithm for certainty rule-based knowledge, integrated group decision rulings and the weighted average theory. In the algorithm, “reliability factor” refers to the reliability of the knowledge in which there is any conflict, overlapping or inconsistency in value. For the conflicting or redundant knowledge, the knowledge of higher reliability can be chosen and extensively applied to the treatment of all certainty rule based conflicting or redundant knowledge. In this way, mistaken decisions can effectively be prevented and more benefits acquired from the knowledge application.

In light of actual needs, the current study presupposes the following restrictions:

1. For the rule-based certain knowledge in the same field, the knowledge conflict treatment is executed.
2. The knowledge in an explored field has been sorted by the knowledge engineer and represented in the rule-based knowledge format characterized by IF (antecedent) THEN (consequent).
3. The explored knowledge conflict treatment refers to the treatment of redundancy rules and conflicting rules. Dead-end rules, accessible result rules and circular rules are not categorized into the knowledge conflict in the current study.

3. Conflicting treatment model for certainty rule-based knowledge

In certainty rule-based knowledge, as it applies to the antecedents or consequents explored in the present study, there exist various overlapping rules or conflicting rules as illustrated in Fig. 1. This study proposes the conditional probability knowledge similarity algorithm and calculation system. The system can quickly and accurately calculate rule-based knowledge similarity matrices and determine the conflicting or overlapping rules. According to the group decision idea, to provide a “reliability factor” refers to the reliability level of the knowledge with conflict, redundancy or inconsistency in value, and to the construction of the Certainty Rule Based Knowledge Conflicting Treatment Model (CRKCTM). In this model, first we use the conditional probability knowledge similarity algorithm to obtain rule-based knowledge similarity matrices, and determine the conflicting or overlapping rules; second we use the “reliability factor” to determine the reliability level of the knowledge with conflict, redundancy or inconsistency in value. The related theories are expanded as follows.

3.1 Conditional probability knowledge similarity algorithm

To comply with the orientation of practical applications, the present research observes the following limiting conditions:

1. It aims at single rule-based deterministic domain knowledge to discuss the similarity between two sets of knowledge; the similarity of knowledge of different domains is not included in this study.
2. The domain knowledge involved in this study has been arranged by knowledge engineers as the representation of rule-based knowledge, characterized by the logic: “IF (antecedent). THEN (consequent)”.
3. The assessment of the knowledge similarity in question refers to the relative similarity between two sets of knowledge after comparison.
4. The minimal distance similarity is zero where occur the maximal of the whole distance.

3.1.1 Knowledge representation in the RO—RA—RV format

The “IF (antecedent) THEN (consequent)”, logic sequence of both antecedent and consequent can be expressed as a sentence or several sentences. A sentence can be expressed in the RO—RA—RV format, comprised of the following four components: the relationship operator (R), the object (O), the attribute (A) and the linguistic value (V).
Here, symbols of the relationship operator $(R)$ may be include the terms $>, =, <, \geq$ and $\leq$. Examples which can be represented in vector form after proper transformation and mapping into numerical types are shown in Table 2.

If the antecedent or consequent sentences contain the processing of logic operators like AND or OR, they are described as follows:

Described by the logic, “IF $(a_1 \text{ AND } a_2)$, THEN $c_1$”, the antecedent vector is formed with six components, written as, \[ [RO_{a_1} \text{ RA}_{a_1} RV_{a_1}|RO_{a_2} \text{ RA}_{a_2} RV_{a_2}] \].

IF $(a_3 \text{ OR } a_4)$ THEN $c_2$ is first divided into two knowledge operations, IF $a_3$ THEN $c_2$ and IF $a_4$ THEN $c_2$, with their antecedent vectors represented as $[RO_{a_3} \text{ RA}_{a_3} RV_{a_3}]$ and $[RO_{a_4} \text{ RA}_{a_4} RV_{a_4}]$, respectively; after the calculations are completed, they are then merged into the original knowledge form IF $(a_3 \text{ OR } a_4)$ THEN $c_2$.

If the antecedent and consequent vectors are of different dimensions, then they are represented by the maximal dimension between them, with the augmented dimensions reduced to zero and the antecedent and consequent vectors represented with the same dimensions.
3.1.2. Transforming the mapping of RO-RA-RV components

The possible data types of O-A-V components in a knowledge sentence are nominal, ordinal, or interval and ratio, as shown in Table 3. The transforming mapping methods for RO-RA-RV components are same.

(1) When the data type of any component is nominal

Then all relationship operators (R) of the components are the equal symbol, and the transformational mapping is 0 or 1 according to Table 4.

(2) When the data type of any component is ordinal

When all relationship operators (R) of O-A-V components are equal signs, they are directly assigned a specific transforming mapping value between zero and one. The highest grade element in component V is assigned a mapping value of one, the lowest grade element in component V is assigned a mapping value of zero, and elements between the highest and the lowest grade in component V are assigned mapping by Eq. (1). Examples of RV transformation mapping values are shown in Table 5.

\[ V_i = \frac{i - 1}{n - 1}, \quad i = 1, 2, \ldots, n \]  

(1)

n is the total series to distinguish weight V. \( V_i \) is the mapping value of ith grade in component V, when \( i = 1 \) is the highest grade.

Table 3

<table>
<thead>
<tr>
<th>Data type of O-A-V components</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>Car color</td>
</tr>
<tr>
<td>Ordinal</td>
<td>Clothes size</td>
</tr>
<tr>
<td>Interval</td>
<td>Body weight, test score, rainfall</td>
</tr>
</tbody>
</table>

Table 4

<table>
<thead>
<tr>
<th>RO-RA-RV components</th>
<th>Mapping value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Existing identical character</td>
<td>1</td>
</tr>
<tr>
<td>No identical character</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5

<table>
<thead>
<tr>
<th>Grade</th>
<th>Description of ( V )</th>
<th>Transforming mapping values of ( RV )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Super-large, very large, very fast</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Large, fast</td>
<td>0.75</td>
</tr>
<tr>
<td>3</td>
<td>Medium, common speed</td>
<td>0.5</td>
</tr>
<tr>
<td>4</td>
<td>Small, slow</td>
<td>0.25</td>
</tr>
<tr>
<td>5</td>
<td>Super-small, very small, very slow</td>
<td>0</td>
</tr>
</tbody>
</table>

(3) When the data type of any component is interval and ratio

(a) The data type keeps its original numerical value. Furthermore, if all relationship operators (R) of the component are equal signs, the effect is to normalize Eq. (2), so as to map the value of \( RV \) into the range from 0 to 1.

\[ V_{norm} = \frac{V - V_{min}}{V_{max} - V_{min}} \]  

(2)

\( V_{norm} \) is the normalized value of \( V \), ranging between 0 and 1, \( V \) is the value of \( V \) before normalization, \( V_{max} \) is the maximal element in component V, \( V_{min} \) is the minimal element in component V.

(b) If not all the relationship operators (R) are equal signs, the transforming mapping of the RV component is determined by the conditional probability theory.

Climinational probability is the probability that the event B occurs, under the condition that the event A occurs too. Mathematically, this is written as

\[ P(B|A) = \frac{P(B \cap A)}{P(A)} \]

where \( P(B \cap A) \) is the probability that both the events A and B occur, and \( P(A) \) is the probability of the event A.

When conducting transformation mapping for two knowledge weights, the testing case weight is set for occurred conditions, in order to compute the chance of having the same weight for any knowledge base case. Hence, the probability for the testing case is 1, which means that the transformation mapping is 1. The probability for the same weight in any knowledge base case is the transformational mapping of the same weight.

Because weight A in the testing case indicates that the weight has occurred, and the probability of weight A is \( P(A) \), thus the conditional probability \( P(B|A) \) can be expressed by Eq. (3).

\[ P(B|A) = P(B \cap A) \]  

(3)
Definition 1

\( V_{\text{max}} \): the maximal element of the component \( V \)
\( V_{\text{min}} \): the minimal element of the component \( V \)
\( r \): the distributed range of the component \( V \),
\( r = V_{\text{max}} - V_{\text{min}} \)
\( x \): the value of the component \( V \) in testing cases (T)
\( y \): the values of the component \( V \) in knowledge base cases (K).

Here, “max” is the maximal element of the component \( V \) with a value \( r \) further added, and “min” is the minimal element of the component \( V \) with a value \( r \) further subtracted, as shown in Fig. 2.

For example, the calculations of the transformational mapping values of \( RV \) components for conditions \( V > x \), \( V > y \) and \( x > y \) are shown in Fig. 3 and Table 6. Because the probability for weight \( V > x \) in the testing case is \( P(V > x) = 1 \), the transformational mapping for weight \( V > x \) is also 1. The number indicates that weight \( V \) between \( x \) and \( max \) can satisfy all \( V > x \) conditions. The linear length formed by \( V \) values that satisfy \( V > x \) conditions is \( max - x \). As for the same weight \( V > y \) in the knowledge base, the weight \( V \) ranges from \( y \) to \( max \), and the linear length formed by \( V \) values that satisfy \( V > y \) conditions is \( max - y \), as shown in Fig. 2.

In the linear length \( max - y \) formed by all \( V \) values that satisfy the same weight \( V > y \) in a knowledge base, due to \( x > y \), the linear length formed by all \( V \) values that can satisfy \( V > x \) conditions in the testing case is \( max - x \). According to Eq. (3), the ratio between the two linear lengths is \( \frac{max - x}{max - y} \), which is also the probability of the same weight \( V > y \) in a knowledge base, in other words, the transforming mapping of the same weight \( V > y \) as shown in Table 7.

![Fig. 2. The distributed range of the component V.](image)

![Fig. 3. Transformational mapping for x>y, V>x, and V>y.](image)

Table 7

<table>
<thead>
<tr>
<th>Category</th>
<th>Value of ( RV )</th>
<th>Mapping value of ( RV )</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>( V &gt; x )</td>
<td>1</td>
<td>The event ( V &gt; x ) has occurred, so ( P(V &gt; x) = 1 )</td>
</tr>
<tr>
<td>( K )</td>
<td>( V &gt; y )</td>
<td>( \frac{max - x}{max - y} )</td>
<td>The event ( V &gt; y ) occurring given that the event ( V &gt; x ) has occurred, then ( P(V &gt; y</td>
</tr>
</tbody>
</table>

The calculations of the transforming mapping values of \( RV \) component, under the different situations are then shown in Tables 8–10.

For the testing cases of \( x_1 < V < x_2 \), and the knowledge base cases \( y_1 < V < y_2 \), which fall within a certain range, respectively, the calculations of the transformational mapping value of the \( RV \) component are shown in Fig. 4 as well as Tables 11–13, respectively.

A summary of transformation mapping of \( RO-R_4-RV \) components is shown in Table 14.

For weights with the same meaning and attributes, a comparison of content may deliver more meanings. Therefore, the transformation should show similar characteristics. In contrast, weights that do not share similarities are worthless for comparison; thus, the fact that they have

Table 8

<table>
<thead>
<tr>
<th>Value</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V &gt; y )</td>
<td>( V = y )</td>
</tr>
<tr>
<td>( T )</td>
<td>( max - y )</td>
</tr>
<tr>
<td>( V &gt; x )</td>
<td>( max - y )</td>
</tr>
<tr>
<td>( V &lt; y )</td>
<td>( max - y )</td>
</tr>
</tbody>
</table>

Table 9

<table>
<thead>
<tr>
<th>Value</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V &gt; y )</td>
<td>( V = y )</td>
</tr>
<tr>
<td>( T )</td>
<td>1</td>
</tr>
<tr>
<td>( V = x )</td>
<td>0</td>
</tr>
<tr>
<td>( V &lt; x )</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 10

<table>
<thead>
<tr>
<th>Value</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V &gt; y )</td>
<td>( V = y )</td>
</tr>
<tr>
<td>( T )</td>
<td>( max - y )</td>
</tr>
<tr>
<td>( V &gt; x )</td>
<td>( max - y )</td>
</tr>
<tr>
<td>( V = x )</td>
<td>( max - y )</td>
</tr>
<tr>
<td>( V &lt; x )</td>
<td>( max - y )</td>
</tr>
</tbody>
</table>
Table 11
Transforming mapping for RV that falls within a range

<table>
<thead>
<tr>
<th>Value</th>
<th>( K )</th>
<th>( x_1 \leq x_2 &lt; y_2 )</th>
<th>( y_1 &lt; x_2 &lt; y_2 )</th>
<th>( x_1 &lt; x_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>( x_1 \geq y_2 )</td>
<td>0</td>
<td>No ((x_2 &gt; x_1))</td>
<td>No ((x_2 &gt; x_1))</td>
</tr>
<tr>
<td>( y_2 &lt; x_1 \leq y_2 )</td>
<td>1</td>
<td>No ((x_2 &gt; x_1))</td>
<td>No ((x_2 &gt; x_1))</td>
<td></td>
</tr>
<tr>
<td>( y_2 &lt; x_1 )</td>
<td>1</td>
<td>No ((x_2 &gt; x_1))</td>
<td>No ((x_2 &gt; x_1))</td>
<td></td>
</tr>
<tr>
<td>( x_1 &lt; x_2 )</td>
<td>1</td>
<td>No ((x_2 &gt; x_1))</td>
<td>No ((x_2 &gt; x_1))</td>
<td></td>
</tr>
</tbody>
</table>

Table 12
Transforming mapping for RV when \( y_1 = \min \)

<table>
<thead>
<tr>
<th>Value</th>
<th>( K )</th>
<th>( x_1 \leq x_2 &lt; y_2 )</th>
<th>( y_2 &lt; x_2 )</th>
<th>( x_1 &lt; y_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>( x_1 \geq y_2 )</td>
<td>0</td>
<td>No ((x_2 &gt; x_1))</td>
<td>No ((x_2 &gt; x_1))</td>
</tr>
<tr>
<td>( y_2 &lt; x_1 \leq y_2 )</td>
<td>1</td>
<td>No ((x_2 &gt; x_1))</td>
<td>No ((x_2 &gt; x_1))</td>
<td></td>
</tr>
<tr>
<td>( y_2 &lt; x_1 )</td>
<td>1</td>
<td>No ((x_2 &gt; x_1))</td>
<td>No ((x_2 &gt; x_1))</td>
<td></td>
</tr>
<tr>
<td>( x_1 &lt; y_2 )</td>
<td>1</td>
<td>No ((x_2 &gt; x_1))</td>
<td>No ((x_2 &gt; x_1))</td>
<td></td>
</tr>
</tbody>
</table>

Table 13
Transforming mapping for RV when \( x_2 = \max \) and \( y_1 = \min \)

<table>
<thead>
<tr>
<th>Value</th>
<th>( K )</th>
<th>( x_1 \leq y_1 )</th>
<th>( x_1 &lt; y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>( x_2 \geq y_1 )</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( x_2 = y_1 )</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( x_2 &lt; y_1 )</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table 14
Summary table of transforming mapping of RO-RA-RV components

<table>
<thead>
<tr>
<th>Data Type</th>
<th>RO-RA-RV components</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>The nominal's transforming mapping is ( 0 ) or ( 1 )</td>
</tr>
<tr>
<td>Ordinal</td>
<td>The ordinal is directly assigned a specific transforming mapping value between ( 0 ) and ( 1 )</td>
</tr>
<tr>
<td>Interval</td>
<td>When all the relationship operators ( R ) of the component are equal signs</td>
</tr>
<tr>
<td>and ratio</td>
<td>When the data type of V is interval and ratio, but not all the relationship operators ( R ) are the equal sign</td>
</tr>
<tr>
<td></td>
<td>The normalized value of V ranging between ( 0 ) and ( 1 )</td>
</tr>
<tr>
<td></td>
<td>The transforming mapping of the RV component is determined by the conditional probability theory</td>
</tr>
</tbody>
</table>

...completely different characteristics should be revealed after the transforming mapping. The rules of NULL value transformation are described as below:

(a) If the RV component is not NULL in the testing case, then all RV components that are "NULL" in the knowledge base cases are transformed with the mapping value of zero.

(b) If the RV component is NULL in the testing case, then all RV components that are "NULL" are transformed with the mapping value of one, and all RV components that are NOT " NULL" are transformed with the mapping value of zero in the knowledge base cases.

For example, if there are three knowledge sets in the knowledge base with the same antecedent \( a_1 \), the rules are as follows:

R7: IF \( a_1 \) THEN \( V > 190 \)
R8: IF \( a_1 \) THEN \( V = 195 \)
R9: IF \( a_1 \) THEN \( V > 165 \)

The values of the component \( V \) have \( \{190, 195, 165\} \) in knowledge base, then \( V_{max} = 195, V_{min} = 165, r = V_{max} - V_{min} = 195 - 165 = 30, max = 195 + 30 = 225 \) and \( min = 165 - 30 = 135 \). The calculations of the transforming mapping values of the RV component are shown from Tables 15–19.

3.1.3. Knowledge similarity calculation

After proper transformational and normalization for the antecedent and consequent knowledge, they may be written in the RO-RA-RV format and represented with vectors.

\( m \) rules represented with an \( n \)-dimensional antecedent and a one-dimensional consequent are combined into a single knowledge matrix as shown in Eq. (4):

Table 15
The RV component for R7 and R9 subject to R7

| Category | Rule no. | Value of RV | Value of \( x \) or \( y \) |
|----------|----------|-------------|----------------|----------------|---------|
| Testing case (\( T \)) | R7 | >190 | \( x = 190 \) |
| Knowledge base cases (\( K \)) | R8 | =195 | \( y = 195 \) |
| R9 | >165 | \( y = 165 \) |

Table 16
RV component transforming mapping for R8 and R9 subject to R7

<table>
<thead>
<tr>
<th>Category</th>
<th>Rule no.</th>
<th>Mapping value of RV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>R7</td>
<td>1</td>
</tr>
<tr>
<td>( K )</td>
<td>R8</td>
<td>1</td>
</tr>
<tr>
<td>R9</td>
<td>0.583</td>
<td></td>
</tr>
</tbody>
</table>

Table 17
RV component transforming mapping for R7 and R9 subject to R8

<table>
<thead>
<tr>
<th>Category</th>
<th>Rule no.</th>
<th>Mapping value of RV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>R8</td>
<td>1</td>
</tr>
<tr>
<td>( K )</td>
<td>R7</td>
<td>0.0286</td>
</tr>
<tr>
<td>R9</td>
<td>0.0167</td>
<td></td>
</tr>
</tbody>
</table>
Table 18
RV component transforming mapping for R7 and R8 subject to R9

<table>
<thead>
<tr>
<th>Category</th>
<th>Rule no.</th>
<th>Mapping value of RV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>R9</td>
<td>1</td>
</tr>
<tr>
<td>$K$</td>
<td>R7</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>R8</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 19
The transforming matrix of RV component of three knowledge sets

<table>
<thead>
<tr>
<th>Rule no.</th>
<th>R7</th>
<th>R8</th>
<th>R9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>0.583</td>
</tr>
<tr>
<td>R8</td>
<td>0.0286</td>
<td>1</td>
<td>0.0167</td>
</tr>
<tr>
<td>R9</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

$K$ (Knowledge Matrix): $K = [k_{ij}]_{m(i+j)}$ with $i = 1, \ldots, m$, and $j = 1, \ldots, (f + m)$

\[
K = \begin{bmatrix}
    a_{11} & a_{12} & \cdots & a_{1l} & c_{11} & c_{12} & \cdots & c_{1l} \\
    a_{21} & a_{22} & \cdots & a_{2l} & c_{21} & c_{22} & \cdots & c_{2l} \\
    \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
    a_{m1} & a_{m2} & \cdots & a_{ml} & c_{m1} & c_{m2} & \cdots & c_{ml}
\end{bmatrix}
\]  

(4)

where $\bar{k}_m = [k_{m1} \ k_{m2} \ \ldots \ k_{m(i+j)}]$, which represents the knowledge row vectors of the $n$th knowledge representation.

When rule-based knowledge is represented as knowledge vectors, such as in $k_i = (k_{i1}, k_{i2}, k_{i3}, \ldots, k_{i(m+i)})$ and $k_j = (k_{j1}, k_{j2}, k_{j3}, \ldots, k_{j(m+j)})$, their Euclidean distance, length, and inner product are defined in Eqs. (5)–(7), respectively.

Euclidean Distance $= D(\bar{k}_i, \bar{k}_j) = \|\bar{k}_i - \bar{k}_j\|$

\[
= \sqrt{\sum_{i=1}^{m(i+j)} [k_{ji} - k_{ij}]^2}
\]  

(5)

Length $= \|\bar{k}_i\| = \sqrt{\sum_{i=1}^{m(i+j)} [k_{ji}]^2}$

\[
= \sum_{i=1}^{m(i+j)} [k_{ji}]^2
\]  

(6)

Inner Product $= (\bar{k}_i, \bar{k}_j) = \sum_{i=1}^{m(i+j)} k_{ji}k_{ij}$

\[
= \sum_{i=1}^{m(i+j)} k_{ji}k_{ij}
\]  

(7)

The present research proposes a knowledge similarity (KKS), which is simpler than the one in Zhang & Korfhage (1999) and much easier to understand. It is the multiplicity between the Distance Similarity (DS) and Angle Similarity (AS). KKS between two knowledge representations, $i$ and $j$, with $m$ rules, will have $m^2$ number of pairwise calculations, which are described in the following equations:

Distance Similarity $= DS(\bar{k}_i, \bar{k}_j) = 1 - \frac{D(\bar{k}_i, \bar{k}_j)}{\max_{n \neq j} D(\bar{k}_i, \bar{k}_j)}$

\[
= 1 - \frac{D(\bar{k}_i, \bar{k}_j)}{D(\bar{k}_i, \bar{k}_j)}
\]  

(8)

Angle Similarity $= AS(\bar{k}_i, \bar{k}_j) = \frac{(\bar{k}_i, \bar{k}_j)}{\|\bar{k}_i\| \|\bar{k}_j\|} = \cos \theta$

\[
= \frac{(\bar{k}_i, \bar{k}_j)}{\|\bar{k}_i\| \|\bar{k}_j\|}
\]  

(9)

Similarity of Knowledge $= KKS_{ij} = DS(\bar{k}_i, \bar{k}_j) \cdot AS(\bar{k}_i, \bar{k}_j)$

\[
= DS(\bar{k}_i, \bar{k}_j) \cdot AS(\bar{k}_i, \bar{k}_j)
\]  

(10)

The KKS value should be in the range from 0 to 1 since both the Distance Similarity (DS) and Angle Similarity (AS) range from 0 to 1. In the same way, the pairwise knowledge similarity can be described by the following equations:

Antecedent Similarity $= AAS_{ij} = DS(\bar{a}_i, \bar{a}_j) \cdot AS(\bar{a}_i, \bar{a}_j)$

\[
= DS(\bar{a}_i, \bar{a}_j) \cdot AS(\bar{a}_i, \bar{a}_j)
\]  

(11)

Consequent Similarity $= CCS_{ij} = DS(\bar{c}_i, \bar{c}_j) \cdot AS(\bar{c}_i, \bar{c}_j)$

\[
= DS(\bar{c}_i, \bar{c}_j) \cdot AS(\bar{c}_i, \bar{c}_j)
\]  

(12)

Antecedent-Consequent Similarity $= ACS_{ij} = DS(\bar{a}_i, \bar{c}_j) \cdot AS(\bar{a}_i, \bar{c}_j)$

\[
= DS(\bar{a}_i, \bar{c}_j) \cdot AS(\bar{a}_i, \bar{c}_j)
\]  

(13)

Consequent-Antecedent Similarity $= CAS_{ij} = DS(\bar{c}_i, \bar{a}_j) \cdot AS(\bar{c}_i, \bar{a}_j)$

\[
= DS(\bar{c}_i, \bar{a}_j) \cdot AS(\bar{c}_i, \bar{a}_j)
\]  

(14)

The Knowledge Similarity Matrix (KSM) is constructed as follows:

\[
KSM = [KKS_{ij}]_{m(i+j)} = \begin{bmatrix}
    1 & KKS_{i2} & \cdots & KKS_{im} \\
    KKS_{21} & 1 & \cdots & KKS_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    KKS_{m1} & KKS_{m2} & \cdots & 1
\end{bmatrix}
\]  

(15)

In the same way, the pairwise knowledge similarity matrix can be described by the following equations:

Antecedent Similarity Matrix $= ASM = [AAS_{ij}]_{m(i+j)}$

Consequent Similarity Matrix $= CSM = [CCS_{ij}]_{m(i+j)}$

Antecedent Consequent Similarity Matrix $= ACSM = [ACS_{ij}]_{m(i+j)}$

Consequent Antecedent Similarity Matrix $= CASM = [CAS_{ij}]_{m(i+j)}$

(16) - (19)

Based on the above discussions, the conditional probability is $P(B|A) = \frac{P(B \cap A)}{P(A)}$ and $P(A|B) = \frac{P(B \cap A)}{P(B)}$ under $P(A) = 1$ may not equal $P(B \cap A)$ under $P(B) = 1$. Therefore, the knowledge matrix formed by the knowledge vectors after the transformation mapping of two equal knowledge weights could be asymmetrical. The similarity matrix obtained by using the asymmetrical knowledge matrix may not be symmetrical.
For example, RV component transforming mapping for a knowledge component conditions \( x > y \), \( V > x \) and \( V > y \) is shown in Fig. 2:

\[
\text{If } P(V > x) = 1 \text{ then } P(V > y|V > x) = \frac{\max x}{\max y} \quad (20)
\]

\[
\text{If } P(V > y) = 1 \text{ and } x > y \text{ then } P(V > x|V > y) = 1 \quad (21)
\]

From Eqs. (20) and (21), we obtain \( P(V > y|V > x) \) which may not equal \( P(V > x|V > y) \). Thus the transforming matrix of the RV component may not be symmetrical.

3.1.4. Conditional probability knowledge similarity algorithm (CPKSA)

To sum up, the present research proposes, as an integration of knowledge engineering, an improved RO-RA-RV knowledge representation compared with the traditional O-A-V one. It does so by employing conditional probability, vectors and matrices, and artificial intelligent rule-based reasoning, and by building a conditional probability knowledge similarity algorithm (CPKSA).

Conditional probability knowledge similarity algorithm

**Input**

- RO, RA and RV //components of m rules
- \( m \) //total numbers of rule
- \( t \) //the maximal dimension of the antecedent vectors
- \( n \) //the maximal dimension of the consequent vectors

**Output**

- ASM, CSM and KSM //Antecedent Similarity Matrix (ASM), Consequent Similarity Matrix (CSM), Knowledge Similarity Matrix (ASM).

**Step 1:** If the antecedent and consequent vectors are in different dimensions then they are represented by the

\[
\text{RF}_i = \frac{\text{The number of the rules with the same antecedent and corresponding consequent in } i \text{th interval}}{\text{The number of the rules with the same antecedent}}
\]

maximal dimension between them, with the augmented dimensions reduced to zero.

\[
\text{The reliability factor of the knowledge with the same consequent}
\]

\[
\text{RF}_i = \frac{\text{The number of the rules with the same consequent and corresponding consequent in } i \text{th interval}}{\text{The number of the rules with the same consequent}}
\]

**Step 2:** According to the data type of components O-A-V and the symbol of the relationship operators (R), do the testing case and knowledge base cases pairwise; transforming mapping so that the values are between 0 and 1; meanwhile they can be presented as numerical knowledge vectors.

**Step 3:** Calculate the length of each knowledge vector, the distance and inner product of any two knowledge vectors, and decide the maximum value of the whole distance among.

**Step 4:** Compute the similarities of the distance, the angle, the antecedent, the consequent and the knowledge for pairwise knowledge vectors.

**Step 5:** Output Antecedent Similarity Matrix (ASM), Consequent Similarity Matrix (CSM) and Knowledge Similarity Matrix (ASM).

**Step 6:** End

5.2. Reliability factor theory

In rule-based knowledge, the treatment of diversified redundant rules or conflicting rules is dependent on the group decision and the concept of resolving disputes and conflicts to reach the intersection, namely a goal universally recognized by each group member. In this paper, reliability factor (RF) refers to redundant rules or conflicting rules and clearly manifests the reliability of each rule. For instance, if three experts suggest three different treatments to the same problems mentioned in Fig. 5, the final treatment shall be supported unanimously by three experts; that is to say, the final treatment is the intersection of three experts. The scheme executed in this way can receive approval from three experts and therefore enjoy the highest reliability.

For knowledge sets which are conflicting, redundant or inconsistent in corresponding value, the way to represent the reliability factor to accurately indicate reliability is put forward as follows.

**Definition 2.** The reliability factor of the knowledge with the same antecedent

\[
\text{RF}_i = \frac{\text{The number of the rules with the same antecedent and corresponding consequent in } i \text{th interval}}{\text{The number of the rules with the same antecedent}}
\]

Additionally, \( 0 \leq \text{RF} \leq 1 \).

According to Eqs. (22) and (23), both bounded intervals and unbounded intervals are included in the distribution interval of the antecedent or consequent of the knowledge
set which is conflicting, overlapping or inconsistent in the corresponding value. Considering that the distribution interval of a rule may have several different reliability factors, we shall discuss the ways to calculate the reliability factor of the rule in the following paragraphs.

(1) When the distribution interval of the antecedent or consequent value of a rule is a bounded interval

When the distribution interval of the antecedent or consequent of a rule falls into a bounded interval, it means that the linear strength of the bounded interval is fixed. Suppose the rule has three different but constant reliability factors, RF₁, RF₂ and RF₃, and their corresponding linear lengths are L₁, L₂ and L₃, respectively, as shown in Fig. 6. In accordance with a weighted average (WA) theory, the RF for the rule can be determined by Eq. (24). If there is a corresponding RF of a fixed point in an interval but the point has no length, then the RF for the point cannot be calculated through Eq. (24); rather it just represents the RF for the value of the point. At the moment, Eq. (24) will be rewritten into the universal representation, as in Eq. (25).

RF = \frac{L₁ \cdot RF₁ + L₂ \cdot RF₂ + L₃ \cdot RF₃}{L₁ + L₂ + L₃} \tag{24}

RF = \frac{\sum_{i=1}^{n} RF_i \cdot L_i}{\sum_{i=1}^{n} L_i} \tag{25}

RFᵢ is the reliability factor of the iᵗʰ interval of the antecedent or consequent value, Lᵢ is the linear length of the iᵗʰ interval of the antecedent or consequent value, n is the total number of the distribution intervals of the antecedent or consequent values, RF is reliability factor for the rule.

(2) When the distribution interval of the antecedent or consequent value of a rule is an unbounded interval or two unbounded intervals

(a) When the distribution interval of the antecedent or consequent value of a rule is an unbounded interval, it means the linear length of an interval in the unbounded interval is indefinitely long. Suppose the rule has three different but constant reliability factors, RF₁, RF₂ and RF₃, and their corresponding linear lengths are L₁, L₂ and L₃, respectively, as shown in Fig. 7, wherein L₁ and L₂ are constants while x is a variable. Through Eq. (25), the RF for the rule can be calculated, as in Eq. (26).

RF = \frac{RF₁ \cdot L₁ + RF₂ \cdot L₂ + RF₃ \cdot \lim_{x \to \infty} x}{L₁ + L₂ + \lim_{x \to \infty} x} \tag{26}

The simplified RF in Eq. (26) approximates RFᵢ, as is shown in Eq. (27). The RF that represents the reliability factor of the rule is approximate to the RFᵢ of an indefinite long interval in the interval.

RF = \frac{RF₁ \cdot \lim_{x \to \infty} x + RF₂ \cdot \lim_{x \to \infty} x + RF₃ \cdot \lim_{x \to \infty} x}{\lim_{x \to \infty} x + \lim_{x \to \infty} x + \lim_{x \to \infty} x} + 1

\approx \frac{RF₁}{1} = RF₁ \tag{27}

(b) When the distribution interval of the antecedent or consequent value of a rule is two unbounded intervals, it means the linear length of two intervals in the unbounded interval is indefinitely long. Suppose the rule has three different but constant reliability factors RF₁, RF₂ and RF₃, and their corresponding linear lengths are L₁, x and y, respectively, as is shown in Fig. 8, wherein L₁ is a constant while x and y are variables. Through Eq. (25), the RF for the rule can be calculated, as in Eq. (28).

Fig. 6. When the distribution interval of the antecedent or consequent value is a bounded interval.

Fig. 7. When the distribution interval of the antecedent or consequent value of a rule is an unbounded interval.

Fig. 8. When the distribution interval of the antecedent or consequent value of a rule are two unbounded intervals.
\[ RF = \lim_{x \to -\infty} x + \lim_{x \to \infty} y \]

(28)

Let \( y = -x \) and transform the variable to get \( \lim_{x \to -\infty} y = -\lim_{x \to \infty} x \). The RF in Eq. (28) can be simplified to get the approximate RF, as in Eq. (29). The RF that represents the rule is approximate to the mean value of the reliability factors of two indefinitely long intervals in the interval.

\[ RF = \frac{RF_1 \cdot L_1 + RF_2 \cdot \lim x + RF_3 \cdot \lim x}{L_1 + \lim x + \lim x} \]

(29)

It can be seen from Eqs. (27) and (29) that when the distribution interval of the antecedent or consequent value of a rule has an unbounded interval, then the reliability factors of other intervals of limited length in the rule are absorbed into the reliability factors of the indefinitely long intervals. The complete failure to affect the actual effect will reduce the precision of the RF for the rule, so it is necessary to fill up the inadequacy.

The main purpose of treating knowledge sets containing conflicts, overlapping or inconsistency in corresponding values is to choose the rule of the highest reliability derived from the knowledge set. The reliability factor is represented by the relative comparison. Thereby the length of extension in the unbounded interval is restricted and the unbounded interval in which the antecedent or consequent value of a rule is distributed is corrected into two bounded intervals which extend themselves in one direction (positive or negative) or both positive and negative directions. Although the extension length is restricted, the reliability factor for the indefinitely long interval will exert the greatest influence over the reliability factor for the rule.

The distribution interval in a rule may have various kinds of reliability factors, and distribution intervals of different types as well. In addition, there is a need to correct the reliability factors under circumstances where operators such as AND or OR are used. The RFs will be specified individually as follows:

**Definition 3**

- \( V_{\text{max}} \): the maximal value of the antecedent or consequent of the knowledge set
- \( V_{\text{min}} \): the minimal value of the antecedent or consequent of the knowledge set
- \( r \): the distribution interval of the previous values of antecedent or consequent of the knowledge set, namely, \( r = V_{\text{max}} - V_{\text{min}} \)
- \( \text{max} \): the values of \( r \), the definite length of the \( V_{\text{max}} \) extension in the positive direction, namely, \( \text{max} = V_{\text{max}} + r \)
- \( \text{min} \): the values of \( r \), the definite length of the \( V_{\text{max}} \) extension in the negative direction, namely, \( \text{min} = V_{\text{min}} - r \)

Based on these definitions, the maximal possible distribution interval of the value of antecedents or consequents of a rule can be shown in Fig. 9.

(3) When the distribution interval of the antecedent or consequent value of a rule extends in positive or negative direction and becomes an interval of definite length

If the distribution interval of the antecedent or consequent value of a rule extends in a positive or negative direction (here we take the extension in the positive direction for example), suppose a rule has three differently constant reliability factors \( RF_1, RF_2 \) and \( RF_3 \) and the linear lengths of these constants are \( L_1, L_2 \) and \( x \), respectively, as shown in Fig. 10. According to Eq. (25), the correction to the reliability factor of the rule can be calculated as follows:

\[ RF = \frac{RF_1 \cdot L_1 + RF_2 \cdot L_2 + RF_3 \cdot x}{L_1 + L_2 + x} \]

(30)

(4) When the distribution interval of the antecedent or consequent value of a rule extends both in positive and negative directions and becomes a bounded interval

If the distribution interval of the antecedent or consequent value of a rule extends in both positive and negative directions, suppose a rule has three differently constant reliability factors \( RF_1, RF_2 \) and \( RF_3 \), and the linear lengths of these constants are \( L_1, x \) and \( y \), respectively, as shown in Fig. 11. According to Eq. (25), the reliability factor of the rule can be calculated as follows:

\[ RF = \frac{RF_1 \cdot L_1 + RF_2 \cdot x + RF_3 \cdot y}{L_1 + x + y} \]

(31)

**Fig. 9.** The maximal distribution interval of the value of antecedent or consequent of a rule.

**Fig. 10.** The distribution interval of the antecedent or consequent value of a rule extends in the positive direction.

**Fig. 11.** When the distribution interval of the antecedent or consequent value of a rule extends both in positive and negative directions.
(5) When the antecedent or consequent value of a rule is connected with the operator AND or OR.
When the antecedent or consequent value of a rule is connected with the operator AND or OR, and the respective reliability factors of $V_1$ and $V_2$ are constants, $RF_1$ and $RF_2$, the RF of the rule can be calculated separately as in Eqs. (32) and (33) (Zadeh, 1992):

\[ R10: \text{IF } a_1 \text{ THEN } (V = V_1 \text{ AND } V = V_2) \]

\[ R11: \text{IF } a_1 \text{ THEN } (V = V_1 \text{ OR } V = V_2) \]

The RF of R10: $RF = \min[RF_1, RF_2]$ (32)
The RF of R11: $RF = \max[RF_1, RF_2]$ (33)

Instance 1: The following five rules are rules with the same antecedent but the corresponding consequent varies from one to another:

\[ R12: \text{IF } a_1 \text{ THEN } V > 42 \]
\[ R13: \text{IF } a_1 \text{ THEN } V > 30 \]
\[ R14: \text{IF } a_1 \text{ THEN } V > 52 \]
\[ R15: \text{IF } a_1 \text{ THEN } V < 60 \]
\[ R16: \text{IF } a_1 \text{ THEN } V = 50 \]

In these five rules with the same antecedent, the distribution interval of the consequent values is an unbounded interval. The values are 42, 30, 52, 60, and 50, respectively, of which $V_{\text{max}} = 60$, $V_{\text{min}} = 30$, then $r = V_{\text{max}} - V_{\text{min}} = 60 - 30 = 30$, $\max = 60 + 30 = 90$, $\min = 30 - 30 = 0$, as shown in Fig. 12.

Looking at R12 for example, R12 has three different reliability factors 0.6, 0.8, and 0.6 and the linear lengths of these constants are 10, 8, and 30, respectively. According to Eq. (30), the RF of this rule is 0.633. By following the same method the reliability factors of other rules can be calculated as follows:

\[ R12: \text{IF } a_1 \text{ THEN } V > 42 \text{ (RF = 0.633)} \]
\[ R13: \text{IF } a_1 \text{ THEN } V > 30 \text{ (RF = 0.587)} \]
\[ R14: \text{IF } a_1 \text{ THEN } V > 52 \text{ (RF = 0.642)} \]
\[ R15: \text{IF } a_1 \text{ THEN } V < 60 \text{ (RF = 0.387)} \]
\[ R16: \text{IF } a_1 \text{ THEN } V = 50 \text{ (RF = 0.8)} \]

Instance 2: The following six rules are ones with the same antecedent; the consequents are different in value and include logic operator AND and OR:

\[ \text{R17: IF } a_1 \text{ THEN } V > 42 \]
\[ \text{R18: IF } a_1 \text{ THEN } 30 < V \leq 50 \]
\[ \text{R19: IF } a_1 \text{ THEN } (V < 20 \text{ AND } V > 70) \]
\[ \text{R20: IF } a_1 \text{ THEN } V < 60 \]
\[ \text{R21: IF } a_1 \text{ THEN } V > 50 \]
\[ \text{R22: IF } a_1 \text{ THEN } (V = 48 \text{ OR } V > 56) \]

The distribution interval of the consequent values of these six rules with the same antecedent is an unbounded interval, in which $V_{\text{max}} = 70$ and $V_{\text{min}} = 20$, $r = V_{\text{max}} - V_{\text{min}} = 70 - 20 = 50$, $\max = 70 + 50 = 120$, $\min = 20 - 50 = -30$ as in Fig. 13.

Looking at R18 for example, the RF of the bounded intervals $(30 < V \leq 50)$ are 0.33, 0.5 and 0.5 (the corresponding linear lengths are 12, 6, and 2). According to Eq. (25), the RF for the rule as a whole is 0.44. In the bounded interval of R19, if there exist two RFs, 0.5 and 0.33, with the corresponding linear lengths 50 and 50, then the RF for the rule as a whole is 0.415. According to Eq. (33), the RF of R22 is max [0.67, 0.473] = 0.667. In the same way, in accordance with Eqs. (25)-(33), the RFs for other rules are calculated as follows:

\[ R17: \text{IF } a_1 \text{ THEN } V > 42 \text{ (RF = 0.466)} \]
\[ R18: \text{IF } a_1 \text{ THEN } 30 < V \leq 50 \text{ (RF = 0.44)} \]
\[ R19: \text{IF } a_1 \text{ THEN } (V < 20 \text{ AND } V > 70) \text{ (RF = 0.415)} \]
\[ R20: \text{IF } a_1 \text{ THEN } V < 60 \text{ (RF = 0.336)} \]
\[ R21: \text{IF } a_1 \text{ THEN } V > 50 \text{ (RF = 0.667)} \]
\[ R22: \text{IF } a_1 \text{ THEN } (V = 48 \text{ OR } V > 56) \text{ (RF = 0.667)} \]

Instance 3: For the following four rules with the same antecedent, there exist conflicts and redundancy between the consequents of these rules:

\[ R23: \text{IF } a_1 \text{ THEN } V > 40 \]
\[ R24: \text{IF } a_1 \text{ THEN } V < 40 \]
\[ R25: \text{IF } a_1 \text{ THEN } V > 40 \]
\[ R26: \text{IF } a_1 \text{ THEN } V > 40 \]

According to Eq. (2), the RF of each rule is as follows:

\[ R23: \text{IF } a_1 \text{ THEN } V > 40 \text{ (RF = 0.75)} \]
\[ R24: \text{IF } a_1 \text{ THEN } V < 40 \text{ (RF = 0.25)} \]

3.3. The calculation of the reliability factor of the combined redundant rules

In Fig. 1, the calculation of the reliability factor of the combined redundant rules is explained as follows.

Instance 4: In Fig. 1a, R25 and R26 are redundant rules:

\[ R25: \text{IF } a_1 \text{ THEN } c_1 \text{ (RF}_{25}\text{)} \]
\[ R26: \text{IF } a_1 \text{ THEN } c_1 \text{ (RF}_{26}\text{)} \]

Because $RF_{25}$ equals $RF_{26}$, the reliability factor of R25 and R26 are merged as $RF_{25}$, and R26 is deleted.
Instance 5: In Fig. 1(b), R27 and R28 are redundant rules:

- R27: IF \((a_1 \text{ OR } a_2)\) THEN \(c_1\) (RF27)
- R28: IF \(a_1\) THEN \(c_1\) (RF28)

If RF27 equals RF28, then directly delete R28; if RF27 is more than RF28, then R27 is modified as follows:

IF \(a_2\) THEN \(c_1\) (RF27).

Instance 6: In Fig. 1(c), R29 and R30 are redundant rules:

- R29: IF \(a_3\) THEN \((c_1 \text{ OR } c_2)\) (RF29)
- R30: IF \(a_3\) THEN \(c_1\) (RF30)

If RF29 equals RF30, then directly delete R30; if RF29 is more than RF30, then R29 is modified as follows:

IF \(a_2\) THEN \(c_2\) (RF29).

Instance 7: In Fig. 1(d), \{R31, R32\} and R33 are redundant rules:

- R31: IF \(a_4\) THEN \(c_1\) (RF31)
- R32: IF \(c_1\) THEN \(c_2\) (RF32)
- R33: IF \(a_4\) THEN \(c_2\) (RF33)

When the minimum of RF31, RF32, and RF33 equals RF31 or RF32, we delete R33. When the minimum of RF31, RF32, and RF31 equals RF31, then RF31 is replaced by RF34, and R31 is deleted.

3.4. Certainty rule-based knowledge conflict treatment algorithm

To sum up, in keeping with the group decision idea, the present research proposes a Certainty Rule-based Knowledge Conflict Treatment Algorithm (CRKCTA), and this approach uses the “reliability factor” to refer to the reliability level of knowledge with conflicts, redundancy or inconsistency in value. Coming up next is a description of CRKCTA.

```
input
ASM, CSM // (Antecedent Similarity Matrix, ASM) and (Consequent Similarity Matrix, CSM) of m rules

Output
RF // the reliability factor of redundant or conflicting rules

Step 1: Select the knowledge sets with redundant or conflicting rules in accordance with the ASM and CSM of m rules.
Step 2: Calculate the reliability factor (RF) for each redundant or conflicting rule among the redundant or conflicting rule sets.
Step 3: Calculate the reliability factor (RF) of the combined redundant rule.
Step 4: Output RF for each redundant or conflicting rule.
Step 5: END

3.5. Certainty rule-based knowledge conflicting treatment model

This study proposes the conditional probability knowledge similarity algorithm to create rule-based knowledge
```

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**Fig. 14. The architecture of CRKCTM.**
similarity matrices and to determine conflicting or overlapping rules, and uses the Certainty Rule-based Knowledge Conflict Treatment Algorithm to refer to the reliability level of knowledge with conflicts, redundancy or inconsistencies in value, and to construct the Certainty Rule-based Knowledge Conflicting Treatment Model (CRKCTM). The architecture of the Certainty Rule-based Knowledge Conflict Treatment Model is shown in Fig. 14.

4. Practical simulation and knowledge application

4.1. Practical simulation

In the practical simulation, 20 rule-based knowledge instances by Huang in 2006 and ASM & CSM were cited. Each rule is designated as in Table 20 (Huang & Cheng, 2006; Huang & Lin, 2005).

Step 1: Select knowledge sets with special relations from the ASM and CSM, as in Table 21; identify the redundant or conflicting knowledge sets, as in Table 22.

Step 2: Figure out the RF for each redundant or conflicting rule among all redundant or conflicting rule sets. A set of redundant rules can be merged into a single rule as in Section 3.3 of this paper, and finally users choose the taller of the RFs for treating conflicting rules, as illustrated in Table 23.

4.2. Knowledge selection and application

Since knowledge reliability is essential to knowledge application, to avoid high risks in knowledge application,

<table>
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<td>Knowledge sets with special relations</td>
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<td>1. Knowledge set with completely the same antecedents</td>
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<tr>
<td>(1) R7, R16, and R19</td>
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<tr>
<td>(2) R15 and R17</td>
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<tr>
<td>(3) R13 and R18</td>
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<tr>
<td>2. Knowledge set with completely the same consequents</td>
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<td>(4) R1 and R9</td>
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<td>(5) R7, R16, and R19</td>
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<td>(6) R12, R15, and R17</td>
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<tr>
<td>3. Knowledge set with a small same knowledge</td>
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<tr>
<td>(8) R7, R16, and R19</td>
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<tr>
<td>(9) R15 and R17</td>
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<td>(10) R13 and R18</td>
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<th>Table 22</th>
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<tr>
<td>Redundant or conflicting rules</td>
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<tr>
<td>Redundant rules</td>
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<td>(1) R7, R16, and R19</td>
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<td>(2) R15 and R17</td>
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<td>(3) R13 and R18</td>
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<td>Conflicting rules</td>
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<tr>
<td>(1) R7, R16, and R19</td>
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<td>(2) R15 and R17</td>
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<td>(3) R13 and R18</td>
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<th>Table 23</th>
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<tr>
<td>RF of redundant or conflicting rules</td>
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<td>Rule type</td>
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<td>Redundant rules</td>
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<td>(1) R7, R16, and R19</td>
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<td>Conflicting rules</td>
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<td>(1) R7 and R20</td>
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<tr>
<td>(2) R12 and R15</td>
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<td>(3) R1 and R9</td>
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the RF of the cited knowledge shall be greater than 0.5. Meanwhile the taller the RF of the cited knowledge is the better. As is mentioned in Table 9, there are three knowledge sets with conflicting rules. They are, respectively, \{R7, R16, R19\}, \{R15, R17\} and \{R13, R18\}. The first set \{R7, R16, R19\} is combined and R16 and R19 deleted. In the same way, the second set deletes R17 and the third set deletes R18. If these combined rules are conflicting rules, then conflict treatment needs to be executed again. Finally we take the RF of conflicting rules, whichever is taller, among R7, R15 and R9.

A questionnaire was sent to average users to gain an understanding of their judgment of the citations of knowledge sets with conflicting or redundant rules, their needs for additional information for auxiliary decisions or knowledge applications, and to check whether the determination of the certainty factor of the conflicting or redundant rules was helpful to their knowledge application or auxiliary decision making. One hundred copies of the questionnaire reached university graduates who were engaged in further research at research institutes or were otherwise employed. Of the 100 copies, 94 were returned for a return rate of 94%. Table 24 illustrates the statistical results of the survey.

5. Conclusions and future prospects

The study came up with four conclusions based on the aforementioned analysis:

(1) The conditional probability knowledge similarity algorithm (CPKSA) and calculation system was proposed. This calculation system can quickly and accurately calculate rule-based knowledge similarity matrices and determine the conflicting or overlapping rules.

(2) The study provided the certainty rule-based knowledge conflict treatment algorithm (CRKCTA) using the “reliability factor,” which refers to the reliability level of the knowledge with conflicts, redundancy or inconsistency in value.

(3) The study combined the CPKSA and CRKCTA, and constructed the certainty rule-based knowledge conflicting treatment model (CRKCTM). This model clearly revealed the RF for conflicting or redundant rules and comprehensively was shown to be useful in gauging the reliability of the cited knowledge. This can then effectively prevent or minimize mistakes in making decisions as a direct benefit to users of the knowledge application.

(4) The questionnaire revealed that 94% of users admitted it was perplexing to cite conflicting or redundant rules; 92% of them held that the definite RF for conflicting or redundant rules was helpful to knowledge applications and auxiliary decision making; and 90% thought the provision of additional relevant and auxiliary information was needed when they were treating conflicting or redundant rules. Moreover, 62% of users said they tended to seek help from others when they were treating conflicting or redundant rules.

On the basis of the existing CRKCTM, our future work will be dedicated to developing a web page-type certainty rule-based knowledge conflict treatment system in order to better the treatment of certainty rule-based knowledge conflicts.
References