Wholesaler Multi-retailer Coordination Policy with Quantity Discount, Price Dependent Demand and Deteriorating Items

S.L. Chung¹, H. M. Wee² and P.C. Yang³

¹Information Management Department, St. John’s University
Tamsui, Taipei 25135, Taiwan, China
²Industrial Engineering Department, Chung Yuan Christian University
Chungli, 32023, Taiwan, China
³Industrial Engineering and Management Department, St. John’s University
Tamsui, Taipei 25135, Taiwan, China
*chung@mail.sju.edu.tw

ABSTRACT
In order to support the collaboration policy, the mutually beneficial relationships have been replacing the traditionally adversarial relationships between wholesaler and retailers. The joint decisions characterized by the ordering price, order quantity and replenishment schedule is collaborated through inventory and pricing policy. This study intends to derive the greatest benefits for both of the wholesalers and the retailers within cooperative environment. The right quantity discounts and the best ordering policy can be achieved by maximizing the joint profit of the deteriorating items under different profit-sharing ratios. We develop a wholesaler-retailers inventory model with an exponentially deteriorating inventory and price dependent demand. A numerical example is used to illustrate theory of the model. The results demonstrate the extra profit accrued mainly to the down-stream site. Thus, profit sharing to derive the greatest benefits for the whole system should be in-cooperated.

Keywords: Wholesaler-sponsored voluntary chains, inventory control, price discount, coordination policy, profit sharing

1. INTRODUCTION
Supply chain management is an integrated approach of planning and controlling materials and information flows. Manufacturers acquire raw materials from the suppliers, make them into finished products, and deliver them to the distributors, then to the retailers. One of the main subjects of the supply chain management is to obtain appropriate mechanisms to coordinate the manufacture, the wholesaler and the retailer in order to maximize the system profit. The joint decision policy is a function of the selling price, the order lot size, the replenishment schedule and the price discount.

Monahan [1] studied the economic implications for the vendor. His analytical approach to vender-oriented optimal quantity discount policy maximized the supplier’s resultant economic gain, but did so at no added cost to the buyer. Lee and Rosenblatt [2] generalized Monahan’s model somewhat on a more general order policy and added a constraint to the discount price. Their algorithm solved the supplier’s joint ordering and price discount policy. Chakravarty and Martin [3] developed a joint profit-sharing scheme between the vendor and the buyer. The algorithm determined both the discount price and the replenishment interval under a periodic review for any negotiation factor. Parlar and Wang [4] considered the supplier’s quantity discount when the supplier’s operating costs are reduced and the buyer’s demand is increased. Weng [5] examined the case where buyer’s demand was a price-sensitive function. Under the general price-sensitive demand functions, Weng [6] developed a model to determine the supplier’s all-unit and incremental quantity discount policies to increase the buyer’s demand and achieve Pareto optimization. Wee [7] extended Chakravarty and Martin’s [3] models to include deterioration and optimal buyer-seller discount pricing and ordering policy with price independent demand.

Recently many researchers have done a lot of research on the discount pricing decision model. Matsuyama [8] modified the EOQ model to consider purchase price discount. Viswanathan and Wang [9] developed the vendor and buyer’s inventory and pricing policy, considering quantity discount and volume discount respectively. Yang and Wee [10] and Yang [11] developed an integrated profit-sharing model between the vendor and the buyer when the demand was price sensitive. An algorithm was developed to determine the end customer’s retail price and the replenishment interval for any desired negotiation factor. Parlar and Weng [12] developed models to study the influence of vertical coordination between the marketing and production division for a company with parallel competition and price-sensitive demand.

The objective of this study is to develop an optimal replenishment and pricing policy by maximizing the retailer and wholesaler’s joint profit in a harmonious cooperation. The study develops a deteriorating inventory model that simultaneously determines the discount price, the number of deliveries and the replenishment interval with exponentially decreasing price dependent demand. By applying mathematical
software, we derive the optimal policy of the system, considering an all-unit discount with one price break. A numerical example with different profit-sharing ratios and negotiation factors is implemented.

2. ASSUMPTIONS AND NOTATION

The mathematical model in this paper is developed on the basis of the following assumptions:

1. The annual demand rate of retailers is exponentially decreasing price dependent.
2. Single-wholesaler and multi-retailer are assumed.
3. Shortage of stock is not allowed.
4. A single item with constant deteriorating rate is considered.
5. There is no replacement or repair for deteriorated units.
6. Holding cost applies to good units only.
7. There are no constraints on space, capacity or capital.
8. The setup cost per run, and the annual holding cost fraction are known and constant.
9. A single-cycle policy is assumed. The wholesaler and retailer order simultaneously.
10. There is integer number of deliveries for each retailer order.

The wholesaler-retailers inventory system is depicted in Figure 1.

The following notations are used:

- \( \theta \) the deterioration rate;
- \( A_w \) the ordering cost for the wholesaler;
- \( A_{ri} \) the ordering cost for the retailer \( i \);
- \( F_w \) the holding cost per dollar per unit time for the wholesaler;
- \( F_{ri} \) the holding cost per dollar per unit time for the retailer \( i \);
- \( P_w \) the unit price for the wholesaler;
- \( p_{ri} \) the unit price for the retailer \( i \) (decision variable);
- \( P_{ri} \) the unit price for the retailer \( i \) before price discount;
- \( p_{ci} \) the retailer \( i \) selling price to the consumer (decision variable);
- \( P_{ci} \) the retailer \( i \) selling price to the consumer before price discount;
- \( a_i \) the retailer \( i \) annual normal demand where there is no price dependent on demand assumption;
- \( b_i \) the ratio of price that influences the annual demand of the retailer \( i \);
- \( d_i \) the annual price-sensitive demand rates of the retailer \( i \) \( (d_i = a_i e^{-b_i p_{ci}}) \);
- \( D_i \) the annual price-sensitive demand rate of the retailer \( i \) before price discount \( (D_i = a_i e^{-b_i P_{ci}}) \);
- \( M \) the number of the retailers that the wholesaler is in charge of;
- \( d \) the annual demand rate of the wholesaler the value is \( \sum_{i=1}^{M} d_i \);
- \( T \) the wholesaler’s ordering cycle time (decision variable);
- \( m_i \) the number of deliveries per \( T \) from the wholesaler to the retailer \( i \) (decision variable);
- \( I_w(t) \) inventory level for the wholesaler when \( t \) is between \( 0 \) and \( T \);
- \( I_{ri}(t) \) inventory level for the retailer \( i \) when \( t \) is between \( 0 \) and \( T/m_i \);
- \( K_i \) the fixed cost to process retailer \( i \) order of any size for wholesaler;
- \( TC_w \) the total relevant cost per unit time for the wholesaler;
- \( TC_{ri} \) the total relevant cost per unit time for the retailer \( i \);
- \( TP_w \) the total profit per unit time for the wholesaler;
- \( TP_r \) the total profit per unit time for the retailer;
- \( JP(P_{r1}, P_{r2}, ..., P_{rM}, P_{c1}, P_{c2}, ..., P_{cM}, m_1, m_2, ..., m_M, T) \) the joint profit per unit time for the whole system.

The following mathematical model is given:
3. MATHEMATICAL MODELING AND ANALYSIS

Figure 2 illustrates the wholesaler-retailer $i$ inventory system. The solid line denotes the wholesaler’s stock; the narrow dashed line represents the retailer’s stock, and the wide dashed line shows the total stock of the wholesaler and the retailer. The retailer’s stock level is represented by the following differential equations:

$$\frac{dI_i}{dt} + \theta I_i(t) = -d_i, \quad 0 \leq t \leq \frac{T}{m_i}, \quad d_i = a_i e^{-b_i t}$$  \hspace{1cm} (1)

Solve the differential equation (1) and boundary conditions $I_i(T/m_i) = 0$. The solution of (1) is

$$I_i(t) = \frac{d_i}{\theta} \left( e^{\frac{t}{\theta}} - 1 \right), \quad 0 \leq t \leq T/m_i$$  \hspace{1cm} (2)

The average inventory level (AIL) of the retailer $i$ is

$$AIL_{ri} = m_i \int_0^T I_i(t) dt = m_i \frac{d_i}{T \theta} \left( e^{\frac{t}{\theta}} - 1 \right) - \frac{d_i}{\theta}$$  \hspace{1cm} (3)

The retailer total relevant annual cost function $TC_{ri}$ can be derived as:

$$TC_{ri}(p_{ri}, p_{ci}, m_i, T) = SC_{ri} + HC_{ri} + PC_{ri} = \frac{m_i A_{ri}}{T} + m_i p_{ri} d_i \left( e^{\frac{t}{\theta}} - \frac{T \theta}{m_i} - 1 \right) + m_i p_{ci} d_i \left( e^{\frac{t}{\theta}} - 1 \right)$$  \hspace{1cm} (4)

The first term is the ordering cost, the second term is the holding cost and the third term is the purchasing cost (including the deteriorated cost). The retailer $i$ total profit is the sales revenue minus the retailer $i$ total cost. It is shown as follows:

$$TP_i(p_{ri}, p_{ci}, m_i, T) = p_{ri} d_i - TC_{ri}(p_{ri}, p_{ci}, m_i, T)$$  \hspace{1cm} (5)

The total profit for all retailers can be expressed as follows:

$$TP = \sum_{i=1}^{M} TP_i(p_{ri}, p_{ci}, m_i, T)$$  \hspace{1cm} (6)

In Figure 2, the wide dash line shows the total stock of the wholesaler and the retailer $i$, the total inventory level is

$$I_{wi}(t) = \frac{d_i}{\theta} \left( e^{\frac{t}{\theta}} - 1 \right), \quad 0 \leq t \leq T$$  \hspace{1cm} (7)

The integrated average total inventory level can be derived as follows:

$$AIL_{wri} = \frac{1}{T} \int_0^T I_{wi}(t) dt = \frac{d_i}{T \theta} \left( e^{\frac{t}{\theta}} - 1 \right) - \frac{d_i}{\theta}$$  \hspace{1cm} (8)

Hence, average inventory level of the wholesaler can be determined by subtracting the average retailer inventory level from the average total inventory level. The actual average inventory level of wholesaler can be derived as:

$$AIL_{w} = \sum_{i=1}^{M} (AIL_{wri} - AIL_{ri}) = \sum_{i=1}^{M} \frac{d_i}{T \theta} \left( e^{\frac{t}{\theta}} - m_i e^{\frac{-t}{\theta}} + m_i - 1 \right)$$  \hspace{1cm} (9)

The wholesaler total relevant annual cost function $TC_w$ can be derived as:
The joint annual profit is the sum of the retailers and the wholesaler total annual relevant cost. It is shown as follows:

\[ TP_i(p_1, p_2, p_M, m_1, m_2, m_M) = \sum_{i=1}^{m} m_i p_i \frac{d}{e^{\alpha m} - 1} - TC_w \]

(11)

The joint annual profit is the sum of the retailers and wholesaler total annual profit.

\[ TP_i(p_1, p_2, p_M, m_1, m_2, m_M) = TP_i + TP_w \]

(12)

When there is no price discount \( p_{ri} = P_{ci}, p_{di} = P_{ci}, D = a e^{\beta p_i} \) the total annual profit of retailer \( i \) is

\[ TP_i(m_i, T) = P_i D_i - m_i A_i + \frac{m_i^2 F_i D_i}{\theta} \left( e^{\alpha m_i} - 1 \right) \]

(13)

For predetermined discount prices \( P_{ri} \) and \( P_{di} \), the optimal ordering interval of retailer \( i \), \( T^0/m_i \), can be found by taking the derivative of (13) with respect to \( T/m_i \) and equating the result to zero:

\[ T^0 = \frac{1}{\theta} \frac{\partial}{\partial m_i} \frac{A_i \theta - P_i F_i D_i - P_i D_i \theta}{P_i D_i (F_i + \theta)} \]

(14)

Substituting value

\[ T^0 = \frac{m_i}{\theta} \frac{A_i \theta - P_i F_i D_i - P_i D_i \theta}{P_i D_i (F_i + \theta)} \]

(15)

Equating the first derivatives of equation (15) with respect to \( m_i \) to zero, one has

\[ \frac{\partial TP_i(m_i, m_2, m_M)}{\partial m_i} = 0 \]

(16)

Since the solution of (16) is not a closed form, the optimal variables of \( m_i \), denoted by \( m_i^* \), is derived numerically. With known \( m_1^*, m_2^*, \ldots, m_M^* \), the optimal value \( T^0 \) can be obtained by (14), and the optimal total profit of retailer \( i \) and the wholesaler with no price discount are:

\[ TP_i^* = P_i D_i - m_i^* A_i + \frac{m_i^2 P_i F_i D_i}{\theta} \left( e^{\alpha m_i} - 1 \right) \]

\[ TP_w^* = P_w D_w - m_w^* A_w + \frac{m_w^2 P_w D_w}{\theta} \left( e^{\alpha m_w} - 1 \right) \]

(17)

respectively.

The annual extra profit of retailer \( i \), \( EP_{ri} \) is the annual profit of retailer \( i \) with price discount minus the corresponding optimal annual profit without price discount.

\[ EP_{ri} = TP_{ri} - TP_{ri}^* \]

(19)

and the wholesaler’s annual extra profit \( EP_{wi} \) is

\[ EP_{wi} = TP_{wi} - TP_{wi}^* \]

(20)

We relate \( EP_{ri} \) to \( EP_{wi} \) values as

\[ EP_{ri} = a_i EP_{ri} \quad a_i \geq 0 \]

(21)

\( a_i \) represents an instrument of negotiation. When \( a_1 = a_2 = \ldots = a_M = 0 \), it means all extra profit sharing are accrued to the retailers; when \( a_1 = a_2 = \ldots = a_M = 1 \), it implies that the extra profit sharing is equally distributed. A large \( a_i \) means that profit is accrued mainly to the wholesaler. The optimization problem is stated as

\[ \max \quad JP(p_1, p_2, p_M, m_1, m_2, m_M, T) \]

s. t. \[ EP_{ri} = a_i EP_{ri} \quad a_i \geq 0 \]

(22)
Hence the optimal value $\text{pr}1$, $\text{pr}2$, $\text{pr}M$, $\text{pc}1$, $\text{pc}2$, $\text{pc}M$, $m1$, $m2$, $mM$, and $T$ that maximizes (22). Since it is a constrained nonlinear mixed programming problem, one can derive the optimal value $\text{pr}1$, $\text{pr}2$, $\text{pr}M$, $\text{pc}1$, $\text{pc}2$, $\text{pc}M$, $m1$, $m2$, $mM$, and $T$ by the following procedure:

(a) From (16) and (14), the optimal number of deliveries from wholesaler to the retailers per cycle $m_i$ and the wholesaler’s optimal ordering interval $T$ before price discount can be derived.

(b) For a range of $m_i$ values ($m_i \leq m^*$), derive a set of Karush-Kuhn-Tucker condition. By using mathematical software, Maple 10, and the Fibonacci Search to satisfy the KKT conditions, we can find an optimal $\text{pr}1$, $\text{pr}2$, $\text{pr}M$, $\text{pc}1$, $\text{pc}2$, $\text{pc}M$, and $T$ value for a specified value of $m_i$ and $a_i$. With a known $\text{pr}1$, $\text{pr}2$, $\text{pr}M$, $\text{pc}1$, $\text{pc}2$, $\text{pc}M$, and $T$, we can derive the optimal total joint profit $\text{JP}(\text{pr}1$, $\text{pr}2$, $\text{pr}M$, $\text{pc}1$, $\text{pc}2$, $\text{pc}M$, $m1$, $m2$, $mM$, $T)$.

(c) Derive the optimal value of $m_i$, denoted by $m^*_i$, that satisfies

$$J\text{JP}(p^*_i,p^*_i,m_i,k^*T) \leq J\text{JP}(p_i,p_i,m_i,k^*T) \geq J\text{JP}(p^*_i,p^*_i,m_i,1,T)$$

Hence the optimal value $\text{pr}1$, $\text{pr}2$, $\text{pr}M$, $\text{pc}1$, $\text{pc}2$, $\text{pc}M$, $m1$, $m2$, $mM$, and $T$ are derived. The results are illustrated in the following numerical example.

### 5. Numerical Example

**Example 1** To illustrate the result of the above theory, the parameters used to illustrate the concept are as follows:

- $M = 5$; $a1 = a2 = a3 = a4 = a5 = 500$; $b1 = b2 = b3 = b4 = b5 = 0.1$; $p_1 = p_2 = p_3 = p_4 = p_5 = $160; $a1 = $210, $a2 = $205, $a3 = 200, $a4 = 210, $a5 = $205; $A1 = $50, $A2 = $60, $A3 = $56, $A4 = $80, $A5 = $60; $F_1 = F_2 = F_3 = F_4 = F_5 = 0.35, A1 = A2 = 0.3; K1 = $80, K2 = $100, K3 = $70, K4 = $120, K5 = $60; $P_w = $80; $F_w = 0.2; A_w = $3000; $a1 = a2 = a3 = a4 = a5 = 1.

Derive the values of $\text{pr}1$, $\text{pr}2$, $\text{pr}M$, $\text{pc}1$, $\text{pc}2$, $\text{pc}M$, $m1$, $m2$, $mM$, $T^*$ and $\text{JP}$. What is the percentage of the joint extra profit ($\text{PJEP}$) in this model?

Using the solution procedure in section 4, the following results are obtained:

1. $m1^* = 5.74, m2^* = 5.07, m3^* = 4.29, m4^* = 4.38$, and $T^* = 0.8603$ year.
2. The optimal discounting price for the retailers, $\text{pr}1$, $\text{pr}2$, $\text{pr}M$, $\text{pc}1$, $\text{pc}2$, $\text{pc}M$, $p_1 = $144.40, $p_2 = $144.50, $p_3 = $146.61, $p_4 = $142.92, $p_5 = $146.89.
3. The optimal discounting price for the consumer, $\text{pr}1$, $\text{pr}2$, $\text{pr}3$, $\text{pr}4$, $\text{pr}5 = $192.69, $192.83, $192.67, $191.82, $191.83.$
4. The wholesaler’s optimal ordering cycle time, $T^* = 0.8027$ years.
5. The optimal number of deliveries per $T^*$ from the wholesaler to the retailers, $m1$, $m2$, $m3$, $m4$, $m5 = 2, 2, 2, 2, 3$.
6. The optimal joint profit per unit time for the whole system is $29521.07$.
7. Table 1 shows the results comparing price discount and with no price discount.

<table>
<thead>
<tr>
<th>Relative variable of model</th>
<th>$i$</th>
<th>$m_i$</th>
<th>$p_i$</th>
<th>$p_{ci}$</th>
<th>$\text{TP}_{ri}$</th>
<th>$T$</th>
<th>$\text{TP}_w$</th>
<th>$\text{JP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No price discount</td>
<td>1</td>
<td>5.74</td>
<td>160</td>
<td>210</td>
<td>2395.79</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5.07</td>
<td>160</td>
<td>205</td>
<td>2191.44</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5.70</td>
<td>160</td>
<td>200</td>
<td>1966.15</td>
<td>0.8603</td>
<td>16924.68</td>
<td>27893.78</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4.29</td>
<td>160</td>
<td>210</td>
<td>2266.93</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5.38</td>
<td>160</td>
<td>205</td>
<td>2148.79</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price discount</td>
<td>1</td>
<td>2</td>
<td>141.40</td>
<td>192.69</td>
<td>2667.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
<td>144.50</td>
<td>191.83</td>
<td>2462.66</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
<td>146.61</td>
<td>192.67</td>
<td>2237.36</td>
<td>0.8027</td>
<td>17195.89</td>
<td>29521.07</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2</td>
<td>142.92</td>
<td>191.82</td>
<td>2538.14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>3</td>
<td>146.89</td>
<td>191.83</td>
<td>2420.01</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Percentage joint extra profit ($\text{PJEP}$) 12.36% 1.60% 5.83%

### 6. Conclusion

A mathematical wholesaler multi-retailer coordinated through price discount is developed for an optimal replenishment and pricing policy when the price dependent demand is exponentially decreasing. The optimal price discount and order policy are derived by maximizing the joint profit of the deteriorating items under different profit-sharing ratios. By KKT optimality conditions and mathematical software, we derive the optimal policy of the system when all-unit discounts are considered. The numerical example shows an extra profit of about 5.83% when our strategy is used. Therefore, price discount policy should be incorporated into the wholesaler-retailer management system.
REFERENCES


