Technical Note

Positioning Control of a Novel Thin-Disc Ultrasonic Motor using Fuzzy Sliding-Mode Control*

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ABSTRACT

The thin-disc piezoceramic-driving ultrasonic actuator dedicated to a stepping ultrasonic motor is proposed. The mechanical hysteresis and dead-zone phenomena are automatically compensated by a closed loop servo control, i.e. Fuzzy Sliding-Mode Control (FSMC), for position tracking. The controller design of FSMC has syncretized the fuzzy reasoning and sliding mode control with robust stability and disturbance rejection based on the verification via the trajectories of stair-up-down stepping square waves. Experimental results show that the proposed FSMC possesses the outstanding tracking and positioning capabilities.

Keywords: Ultrasonic motor, Piezoceramic, Nonlinear, Fuzzy sliding-mode

I. INTRODUCTION

Since the cost of accuracy ultrasonic motors (USM) is so high that the commercial one is quite expensive, the thin-disc piezoceramic (PZT) driving ultrasonic actuators have been developed [1-5], recently. The thin-disc PZT is used as an exciting source after conducted a high frequency AC power. The thin-disc actuator applies mechanical vibration of the metal sheet to transfer energy from the electrical power. The PZT can be operated in an ultrasonic frequency with the amplitude of several micrometers, which is controllable by input exciting voltage signal. However, these issues are the lack of descriptions relative to system nonlinear features for the design of a thin-disc ultrasonic actuator. The ultrasonic friction drive exhibits a stick-slip behavior, particularly at the start of running. This phenomenon may be closely relative to positioning errors. In addition, several mechanical features are still difficult to be modeled precisely such as hysteresis. In the recent development, sliding-mode control scheme has been successfully applied to ultrasonic actuators [6, 7]. In this study, sliding-mode control and fuzzy reasoning technology are integrated as an achievement of accuracy and inexpensive method for going beyond such limitations in nonlinear properties of a thin-disc USM. Therefore, the design of an efficient controller for USM operation should be performed in order to minimize the variation of dynamic behaviors and disturbances of the whole system. Fuzzy sliding-mode control (FSMC) will compensate automatically the effect of unmodeled mechanical behaviors on the quality of the positioning motion. If system perturbations are of matching type, the system response in FSMC will be completely insensitive to those perturbations, and then a robust performance can be guaranteed.

II. ACTUATING AND NONLINEAR BEHAVIORS OF A USM

This innovative USM using PZT driving element, with dimension specification as shown in Figure 1 (a) and the prototype of USM constructed as illustrated in Figure 1(b). The rotor gear is rotated with an ultrasonic friction actuator. In operation, after a high frequency AC power input, the PZT will generate oscillating motion as an actuator due to the converse piezoelectric effect, while operating frequency launched over 30 kHz. The generated flexural waves on the metal sheet are transferred along radial or transverse directions. Some
reflecting waves can be formed by screw constraints. By setting three reflecting points on three peripheral constraints in pitched 90°, 120°, 150° angle respectively, as shown in Figure 1(a). Therefore, the driving mechanism of vibration mode for a PZT thin-disc USM was constructed [6, 7].

From ANSYS simulations shown in Figure 1(c) and Figure 1(d), at 90° arc edge region, the deformation vectors have different directions in both of radial (R) and transverse (θ) components when the voltage frequencies of 67 kHz and 75 kHz are individually inputs. The variation of the deformation would be enlarged with the higher voltage. When a rotor is tightly against the driving point, as the frequency is at 67 kHz, the rotor will be driven to rotate in counterclockwise (θR) direction; as the frequency is at 75 kHz, the rotor will move in clockwise (θF) direction. Thus, the contact edge of 90° arc region could turn the rotor in clockwise and counterclockwise directions, respectively, via the metal sheet deflection.

According to the comparisons of Figure 1(c) and Figure 1(d), the combination deflections of 75 kHz in R and θ components are greater than that of 67 kHz. That means the output torque of 75 kHz is larger than that of 67 kHz. When the rotor is placed on the edge of the actuator being 90° arc region, the actuator will rotate the rotor in a state of disequilibrium in clockwise or counterclockwise direction, as shown in Figure 2. By ceaselessly running the motor in clockwise and counterclockwise directions by mean of periodic sinusoidal commands, the home position of geometrical positioning mark on rotary platform was never turned back. Because θR is always less than θF, there are escalating slopes as shown in Figure 2(a). And linear increment amount θF - θR for position errors is continuously multiplied, therefore, the position trajectory of the USM looks like as raised loop with identical pitches, as shown in Figure 2(b).

For the PZT thin-disc vibration in ultrasonic frequency, the PZT is the driving source to generate the high frequency extended-shrunk motion of the metal sheet. The metal sheet plays the amplifier of vibrating magnitude and constructs the desired mode shapes. In contrast, the metal stiffness of the sheet also constrains the PZT, and limits the PZT dynamic response to electric power. Only in suitable power entry matching resonance frequency and appropriate mechanism configuration for rotor-stator contacts, the metal stiffness would be overcome by the PZT to produce the optimal efficiency. However, there is dead-zone phenomenon due to structure hysteresis and frictional stick-slip circumstance, as shown in Figure 3 described the characteristic curve of rotor speeds depending on the applied voltage and driving frequencies. The dead-zone line (a) displays the existence of dead-zone phenomenon under free loading owing to the compound effect of metal deformation and frictional transmission. If voltage amplitude is lower than ± 3.8 V, the extended-shrunk deformation on the metal sheet cannot perform well to obtain the effective mechanical output for rotor turning. Otherwise, when adding external loading onto the platform of the USM, the dead-zone phenomena in rotational characteristic

![Figure 1](image-url) Thin-disc structure ultrasonic motor (a) ultrasonic actuator with its three constraints placed 90°, 120°, 150° arc location and its dimensions ; (b) the prototype ; (c) simulating vibration mode at 67 kHz; (d) simulating at 75 kHz. Arrows point to the constraint points.
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Figure 2 The hysteresis characteristic of rotor position for USM running in clockwise and counterclockwise direction.

(a) Free load
(b) with load 350g
(c) with load 700g
(d) with load 1050g

Figure 3 Rotary speeds versus applied voltage for two specific frequencies of the USM.

III. FSMC DESIGN OF AN ULTRASONIC SERVO DRIVE SYSTEM

To implement an USM servo control system, it is necessary to obtain the dynamic model of the USM mathematically. However, the dynamic model of the USM is extremely difficult to derive from a theoretical point of view because it contains many complicated and nonlinear characteristics, which depend on the operating temperature, load torque, applied voltages, and static pressure force between the stator and the rotor of a USM. Moreover, according to the ultrasonic modal analysis and actuating characteristic curves of rotary speed and driving frequency [7], angle displacement per input signal period is directly determined by the sinusoidal amplitude of input voltage onto the PZT of this ultrasonic motor. And, the rotary direction is controlled by specific driving frequencies of sinusoidal input voltage. In order to satisfy such characteristics of frequency control for an ultrasonic motor, one set of effective driving circuits was implemented. By using the driving circuit [6-8], an experimental model of the USM can be established by system identification technique for the purpose of controller design.

The approximated transfer function of rotor angular speed to actuator input voltage is \( \omega(s)/V(s) = \frac{b}{s+a} \). As such, after time integral, the equation represents the approximated transfer function of rotor position to stator input voltage as \( \theta(s)/V(s) = \frac{b}{s(s+a)} \). The second order transfer function of rotor position to input voltage for the ultrasonic motor is represented in the following mathematical model,

\[
\theta(s) = \frac{k_\theta}{s(1+\tau_s)}
\]

Assuming the non-linearity behavior of ultrasonic system as the equivalent disturbance torque, the differential equation (1) becomes [9],

\[
\dot{\theta}(t) = -a_\theta \dot{\theta}(t) + b_\theta u(t) - \frac{1}{J_0} T_d(t)
\]
\[ T_d(t) = T_L + \Delta \dot{\theta}(t) + J_0 \Delta \dot{a}(t) - J_0 \Delta \dot{b}(t) \quad (2b) \]

where \( T_d(t) \) is an equivalent disturbance torque; \( a_0 = 1/\tau \), \( b_0 = k_m/\tau \); \( J_0 \) and \( T_L \) are an equivalent constant for the moment of inertia and loading torque, respectively. \( \Delta a \), \( \Delta b \), and \( \Delta J \) are corresponding to the varying amount of normalized system parameters \( a_0 \), \( b_0 \), and \( J_0 \), respectively. All system uncertainty amount, \( \dot{d}(t) = T_d(t)/J_0b_0 \), are also assumed to be bounded as follow,

\[ |\dot{d}(t)| \leq k \quad (3) \]

Figure 4 shows the block diagram of USM position control system using FSMC, where \( x_d(t) \) is defined as the desire position and angular displacement command, and \( x(t) \) as the real position and angular displacement of the rotor. In order to achieve smoothing position command, the pre-filter is introduced. And the state variables of system as follows,

\[ x(t) = (x_1(t), x_2(t))^T = (\theta(t), \omega(t))^T \quad (4a) \]

\[ x_1(t) = (\theta_1(t), \omega_2(t))^T \quad (4a) \]

In this case, the tracking error is:

\[ e(t) = x(t) - x_d(t) \quad (5) \]

The system dynamic equation then becomes,

\[ \dot{x}(t) = Ax(t) + Bu(t) + d(t) ; y(t) = Dx(t) \quad (6) \]

where

\[
A = \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ b \end{bmatrix}, \quad D = [1 \ 0]
\]

\[ a = a_0 \pm \Delta a, \ 3.94 \leq a \leq 10.99 \]

\[ b = b_0 \pm \Delta b, \ 1.932 \leq b \leq 13.52 \]

\[ a_0 = 7.465, \ b_0 = 7.726 \]

\[ \Delta a = 3.525, \ \Delta b = 5.794 \]

Their variations are caused by the change of the mass of external loads, the imprecision of parameters and external disturbance. According to above mathematical model, the design for fuzzy sliding-mode controller is as followin section.

### 3.1 Sliding function design

In order to stabilize the system trajectory for approaching the control target \( x=0 \) in sliding mode, it is necessary to choose the appropriate sliding function \( s(t) \) with an integral form of sliding function as follows,

\[ s(t) = c_1e(t) + c_2 \int_0^t e(\tau)d\tau \quad (7) \]

where \( c_1, \ c_2 \in R^{1 \times 2} \) is positive constant matrices and \( c_1B \neq 0 \).

#### 3.2 FSMC position controller design

To assure that the system trajectory approaches \( x(t)=0 \) in limited time and that the sliding mode behavior exists, the position controller is designed as

\[ u(t) = u_{eq}(t) + u_{sw}(t) \quad (8) \]

\[ u_{eq}(t) = (c_1B)^{-1}(-c_1Ax - c_2x + c_1\dot{x}_2) \quad (8.1) \]

\[ u_{sw}(t) = -(k + \sigma)\text{sign}(s(t)) \quad (8.2) \]

where \( u_{eq}(t) \) is the input term of equivalent control \( u_{eq}(t) \) is the input term of discontinuous switching control; and \( \text{sign}(\cdot) \) is a sign function that is an ideal switching function. Definition as follow,

\[ \text{sign}(s(t)) = \begin{cases} +1, & \text{if } s(t) > 0 \\ -1, & \text{if } s(t) < 0 \end{cases} \quad (9) \]

where \( k \) is defined as \( |\dot{d}(t)| \leq k \), \( \sigma > 0 \) is the speed of approaching \( s(t)=0 \) and both of \( c_1B \) and \( k \) are positive values.

**Lemma:** If the sliding function \( s(t) \) of control system satisfied the following conditions, then the existing condition of guaranteed approaching and sliding mode behavior would be sustained [10].

\[ s(t)\dot{s}(t) \leq -\sigma|s(t)| \quad \sigma > 0, \ s(t) \neq 0 \quad (10) \]

Only if \( k \geq |\dot{d}(t)| \), the existing condition of approaching and sliding mode behaviors in lemma would be satisfied by SMC position controller using equation (8). System trajectory shall steadily slide to control target along \( s(t)=0 \). In order to prevent the non-interrupt chattering phenomena in tiny space at sliding mode of system tracking path and unexpected high frequency noise,
bound layer concept promoted by Slotine [11] was employed instead of switching function. That is, \(\text{sign}(s)\) was alternative to \(\text{sat}(s, \varepsilon)\). The function of \(\text{sat}(s, \varepsilon)\) is defined as

\[
\text{sat}(s, \varepsilon) = \begin{cases} 
1 & s > \varepsilon \\
\frac{s}{\varepsilon} & |s| \leq \varepsilon \\
-1 & s < -\varepsilon 
\end{cases}
\] (11)

Where \(\varepsilon\) is infinitesimal positive. For system trajectory reaching \(s(t)=0\) quickly, the corrective term \(\alpha s(t)\) was subjoined into the control rule. Hence, the modified control rule is

\[
u(t) = \frac{1}{2}\left(c_1Ax - c_1x + c_2\dot{x} + c_3\dot{x}_f\right) - (k + \sigma)\text{sat}(s(t), \varepsilon) + \alpha \cdot s(t)
\] (12)

The control input term \(u_{\text{ue}}\) of equation (8) was applied to suppress uncertain amount in system, but the determination of \(k\) value was difficult. If \(k\) value is too much, it will lead control input onto extremely chattering phenomena. If \(k\) value is too less, probably the system will be unstable. Thus, in this study, the inference mechanism of fuzzy control theory was employed to deal with \(k_f\) rather than \(k\) value [12, 13]. The design stages of fuzzy sliding mode control are described as

\[
u(t) = \frac{1}{2}\left(c_1Ax - c_1x + c_2\dot{x} + c_3\dot{x}_f\right) - (k_f \cdot \text{sat}(s(t), \varepsilon) + \alpha \cdot s(t))
\] (13)

where \(k_f\) estimating via reasoning from fuzzy control, as shown in Figure 4. Assigned \(s(t)\) and \(\dot{s}(t)\) as input variables and \(k_f\) as an output variable, \(k_c\) and \(k_s\) are input quantitative factors and \(k_{\alpha k}\) is output quantitative factor, respectively, i.e.,

\[
s_f = s \times k_s \quad ; \quad s_{\dot{f}} = s \times k_s \quad ; \quad k_f = \Delta k_f \times k_{\alpha k}
\]

It is a very important term of \(k_{\alpha k}\) assignment. If \(k_f\) was a relative large positive value, there was an extra high frequency chattering in the system. In contrast, assuming a relative less negative \(k_f\) value, the system trajectory will be far away or the worst case is never reaching the \(s(t)=0\). Therefore, the design of \(k_{\alpha k}\) should be considered carefully.

The value of \(k_f\) would be too less if \(\dot{s}(t)\) is positive so that the influence of term of \(d(t)\) could not be overcome and system trajectory would be far away \(s(t)=0\). Therefore, in this situation, the value of \(k_f\) should be increased to positive gradually. In contrast, if \(\dot{s}(t)\) is negative, the approaching mode was satisfied so that the system motion trajectory would be closer to \(s(t)=0\). The control input obviously became greater resulting in surplus control. Therefore, to avoid undesired surplus control, the value of \(k_f\) should be modified to negative gradually. But the negative value of \(k_f\) should not be too less so that the system would be far away \(s(t)=0\) again. In order to guarantee the whole motion trend towards \(s(t)=0\), the membership of the output signal was designed as the usual functions of Fuzzy sets. Especially, in the case of \(\dot{s}(t) = 0\) and \(s(t) \neq 0\), the value of \(k_f\) should be increased for system trajectory approaching \(s(t)=0\). For accomplishing above fuzzy reasoning, the fuzzy logic rule tables in different status are shown in Table 1 and Table 2. The fuzzy implication rule of Mamdani’s min-min-max method is used for fuzzy reasoning. The output \(k_f\) of fuzzy inference is calculated in defuzzification through the Center of Gravity method.

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<tr>
<th>(SS_f)</th>
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**IV. RESULTS AND DISCUSSION**

For the performance evaluation of controllers, the stair-up-down step signals are the inputs for position tracking commands through the pre-filter, a second order transfer function. It would be \(\xi = 1\), \(\omega_c = 10\), and

\[
m(s) = \frac{m_2}{s^2 + m_1s + m_2} = \frac{\omega^2_s}{s^2 + 2\xi\omega_s s + \omega^2_s}
\] (14)

In control system design, the characteristic roots of system shall be placed in LHP during sliding mode. System trajectory being along \(s(t)=0\) and control target will be gradually reached. If the placement of dual characteristic roots in the design of close loop system is located in (-20, 0) of complex plane, C1 and C2 of sliding function \(s(t)\) in equation (7) are equal to (1,1) and (400, 39), respectively. The parameter \((a, e)\) of controller \(u(t)\) in equation (13) is (25, 0.001).

The ultrasonic motor was experimented at free loading and 1 kg loading, respectively, onto carrying platform of testing bench. The stair-up-down step responses of the rotor position, tracking error, and control input are addressed in the FSMC as shown in Figure 5. The tracking ability is excellent under either free loading or 1 kg loading. Their tracking error is less than 0.001–0.002 radius at small displacement command of 0.01 radius using FSMC method. The control input is affected by
Figure 5  The experimental results with stair-up-down step command: (a), (b), and (c) are rotor position, tracking error, and control input of FSMC (free loading); (d), (e), and (f) are rotor position, tracking error, and control input of FSMC (1 kg loading) \( [r = position \text{ command}; \; x = rotor \text{ position}]. \)

According to tracking errors of external loading analysis in stair-up-down step commands, the tracking error amount of FSMC controller is closer to that of free loading. That is, FSMC controller has efficient performance to overcome the dead-zone phenomena and hysteresis behaviors for this novel USM. And, system external 1 kg loading. And, chattering phenomena is existence either at free loading or at external 1 kg loading.
uncertainties and external disturbance in FSMC could be overall insensitive and robust in noise rejection. As such, the robust performance in noise rejection can be guaranteed. In FSMC system, the input term $u_{sw}$ has the inference mechanism of fuzzy theory in term of $k_f$ of equation (13) rather that $k$ of equation (12) resulting that the dynamic behaviors of control inputs are well-being at the amount of tracking error, but FSMC has little amount in control inputs and less high-frequency chattering phenomena.

V. CONCLUSIONS

When FSMC strategy is employed for position tracking in forward and reverse rotations, the mechanical hysteresis inherently and the dead-zone are obviously eliminated. Evaluating the performance of FSMC strategy in term of position tracking commands with stair-up-down step signals, experimental results prove that robust control and tracking error can be confined by FSMC. The thin-disc ultrasonic servomechanism is suitable for the stepping applications without high frequency chattering and with less control input.

REFERENCES


