Optimal pricing and ordering policy for perishable items with limited storage capacity and partial trade credit

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[Received on 4 May 2011; accepted on 2 February 2012]

Thangam & Uthayakumar (2010, Optimal pricing and lot-sizing policy for a two warehouse supply chain system with perishable items under partial trade credit financing. Oper. Res. Int. J., 10, 133–161) investigated perishable items with limited storage capacity under a partial trade credit policy. For simplification, they used Taylor series approximations to derive their solution procedures. In our paper, we develop an exact solution procedure that avoids the problem of assuming that the deterioration rate is small, thereby improving the accuracy of the solution but more fundamentally increasing the practical relevance of the solution.

Keywords: partial trade credit; EOQ model; supply chain; perishable items.

1. Introduction

Deterioration refers to the damage, spoilage, dryness, vaporization, etc., of products in the supply chain. Inventory problems for deteriorating items have been studied by many researchers. Research in this area started with the work of Whitin (1957) who considered fashion goods deteriorating at the end of a prescribed storage period. An exponentially decaying inventory was developed first by Ghare & Schrader (1963). Since then considerable work has been done on deteriorating inventory systems, the details of which can be found in the review articles of Nahmias (1982), Raafat (1991), Groenevelt et al. (1992), Goyal & Giri (2001) and Balkhi (2004).

Trade credit represents one of the most flexible sources of short-term financing available to firms principally because it arises spontaneously with the firm’s purchases. Goyal (1985) established an economic order quantity (EOQ) under the condition of a permissible delay in payments. He assumed that the supplier would offer the retailer a delay period but the retailer would not offer the trade credit period to customers. Huang (2003) extended Goyal (1985) to provide a fixed trade credit period $M$ between the supplier and retailer and a trade credit period $N$ between the retailer and customers. Many related articles can be found, e.g. Liao & Chung (2009), Min et al. (2010) and Kreng & Tan (2010). Moreover, Chang et al. (2008) have reviewed articles concerned with inventory models under trade credit.

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Once a trade credit has been offered, it can have the effects of stimulating demand from the retailer, thus encouraging the retailer to order large quantities because a delay of payments indirectly reduces inventory costs. Hence, the retailer may purchase more goods than can be stored in his/her own warehouse $W_1$. These excess quantities will be stored in a rented warehouse $W_2$. Furthermore, to reduce default risks, in practice, some suppliers/retailers frequently offer a partial trade credit to retailers/customers who must pay a portion of the purchase amount at the time of placing an order as a collateral deposit and receive a permissible delay on the rest of the outstanding amount. Huang (2005) allowed that the supplier would offer the partial credit but not full trade credit to the retailer. In addition, Huang & Hsu (2008) assumed that the retailer would obtain the full trade credit offered by the supplier and he/she would just offer partial trade credit to his/her customers. Recently, Mahata & Mahata (2011) investigated the partial trade credit policy with two levels of trade credit to reflect supply chain situation in a fuzzy sense. These arguments illustrate that partial trade credit offered by the supplier or retailer can be used in business transactions in practice.

Previous modellers of the EOQ describe two types of demand functions. One is a linear decreasing function of price. Other is a negative power function of price. Datta & Paul (2001) analysed a multiperiod EOQ model with stock-dependent and price-sensitive demand rate. Papachristos & Skouri (2003) extended the EOQ model to allow the demand rate to be a convex decreasing function of the selling price. Sana & Chaudhuri (2008) studied an inventory model with advertising sensitive demand. Both Ouyang et al. (2008) and Ouyang et al. (2009) considered the integrated inventory models with price-sensitive demand rate to be a negative power function of the selling price. Sana (2010, 2011) formulated an EOQ model for deteriorating items while the demand is price sensitive. In addition, Konstantaras et al. (2011) studied the optimal pricing for build-to-order products in a supply chain system. Recently, Thangam & Uthayakumar (2010) further generalized the demand to be a decreasing function of the selling price.

Thangam & Uthayakumar (2010) combined the above ideas to specify an EOQ-based model with perishable items and two-storage facilities as a profit maximization problem with a partial trade credit policy and price-dependent demand. Their total profit per unit time is a function of the selling price and cycle time and involves the exponential term $e^{\theta T}$ in the formulation, where $\theta$ is the deterioration rate and $T$ is the replenishment cycle length. Essentially, their inventory model is correct and interesting. However, they use Taylor series approximations and neglect third- and higher-order terms in the expansion of $e^{\theta T}$ in order to get closed-form solutions, thus simplifying the procedure for locating the optimal solution. Many examples in Chung et al. (2009) reveal that the Taylor series approximations method may yield significant penalty costs in certain circumstances. Hence, there exist reasons and motivations to present an exact solution procedure. This is the main purpose of our paper. In the next section, we formulate the problem. In Sections 3 and 4, we present our results and describe our solution algorithm in Section 5. In Section 6, numerical examples are used to illustrate the accuracy of ours. We conclude with a discussion.

2. Mathematical formulation

The following notations and assumptions will be used throughout the paper.

2.1 Notations

- $A$: Ordering cost per cycle
- $h_1$: Retailer’s holding cost per unit time in $W_1$ excluding interest charges
- $h_2$ ($>h_1$): Retailer’s holding cost per unit time in $W_2$ excluding interest charges
- $c$: Retailer’s procurement cost per unit item
2.2 Assumptions

1. Demand, $\lambda(s)$, is a decreasing function of $s$.

2. The sales revenue, $(s - c)\lambda(s)$, is a concave function of $s$. It stems from the standard economic effect: the marginal revenue decreases as output decreases.

3. The time to deterioration of a product follows an exponential distribution with parameter $\theta$, i.e. the deterioration rate is a constant fraction of the on-hand inventory. It is assumed that the deterioration rate in $W_1$ is the same as in $W_2$.

4. Before the settlement of an account, the retailer can use sales revenue to earn the interest. At the end of period $M$, the credit is settled and the retailer starts paying the capital opportunity cost for the items in stock with an annual rate $I_k$.

5. The retailer offers partial payment scheme at the rate of $\alpha$ to his customer. Then his customer must pay off the remaining balance at the end of period $N$. Hence, the retailer can earn interest with rate $I_e$ for the period of length $M$.

6. Time horizon is infinite.

7. Inventory holding cost is charged only on the amount of undecayed stock.

8. Shortages are not allowed.

Based on the above notations and assumptions, Thangam & Uthayakumar (2010) showed that the annual total profit $TP(s, T)$ can be divided into two cases. One is $M < N$ and the other is $M \geq N$.

Case (A): $M < N$

(A1) When $t_1 \leq M$,

$$TP(s, T) = \begin{cases} 
TP_1(s, T) & \text{if } 0 < T \leq t_1, \\
TP_2(s, T) & \text{if } t_1 < T \leq M, \\
TP_3(s, T) & \text{if } M < T, 
\end{cases} \quad (1a)$$

(A1b) $TP_1(s, T)$

(A1c) $TP_2(s, T)$

(A1d) $TP_3(s, T)$
where

\[
\text{TP}_1(s, T) = (s - c)\dot{\lambda}(s) - \frac{A}{T} - \frac{(c\theta + h_1)\dot{\lambda}(s)}{T\theta^2} [e^{\theta T} - \theta T - 1] + s I_e \alpha \dot{\lambda}(s)[M - T/2],
\]

\[
\text{TP}_2(s, T) = (s - c)\dot{\lambda}(s) - \frac{A}{T} - \frac{(c\theta + h_2)\dot{\lambda}(s)}{T\theta^2} [e^{\theta T} - \theta T - 1] + \frac{(h_2 - h_1)\dot{\lambda}(s)}{\theta^2 T} [(\theta T - \theta t_1 + 1)e^{\theta t_1} - (1 + \theta T)] + s I_e \alpha \dot{\lambda}(s)[M - T/2],
\]

\[
\text{TP}_3(s, T) = (s - c)\dot{\lambda}(s) - \frac{A}{T} - \frac{(c\theta + h_2)\dot{\lambda}(s)}{T\theta^2} [e^{\theta T} - \theta T - 1] - \frac{c I_k \dot{\lambda}(s)}{\theta^2 T} [e^{\theta(T-M)} - \theta(T - M) - 1] + \frac{s I_e \alpha \dot{\lambda}(s) M^2}{2T}.
\]

and

\[
t_1 = t_1(s) = \frac{\ln[1 + \frac{\theta Z}{\alpha(s)}]}{\theta}.
\]

(A2) When \( t_1 > M \),

\[
\text{TP}(s, T) = \begin{cases} 
\text{TP}_1(s, T) & \text{if } 0 < T \leq M, \\
\text{TP}_4(s, T) & \text{if } M < T \leq t_1, \\
\text{TP}_3(s, T) & \text{if } t_1 < T, 
\end{cases}
\]

(B1) When \( t_1 < N \),

\[
\text{TP}(s, T) = \begin{cases} 
\text{TP}_5(s, T) & \text{if } 0 < T \leq t_1, \\
\text{TP}_6(s, T) & \text{if } t_1 < T \leq N, \\
\text{TP}_7(s, T) & \text{if } N < T \leq M, \\
\text{TP}_8(s, T) & \text{if } M < T, 
\end{cases}
\]

where

\[
\text{TP}_5(s, T) = (s - c)\dot{\lambda}(s) - \frac{A}{T} - \frac{(c\theta + h_1)\dot{\lambda}(s)}{T\theta^2} [e^{\theta T} - \theta T - 1] + \frac{s I_e \alpha \dot{\lambda}(s)[M - (1 - \alpha)N - \alpha T/2]}{2T},
\]

Case (B): \( M \geq N \)
\[
TP_6(s, T) = (s - c) \lambda(s) - \frac{A}{T} - \frac{(c\theta + h_2)\lambda(s)}{T\theta^2}[e^{\theta T} - \theta T - 1]
+ sI_e\lambda(s)[M - (1 - \alpha)N - \alpha T/2]
+ \frac{(h_2 - h_1)\lambda(s)}{\theta^2 T}[(\theta T - \theta t_1 + 1)e^{\theta t_1} - (\theta T + 1)],
\]
(9)

\[
TP_7(s, T) = (s - c) \lambda(s) - \frac{A}{T} - \frac{(c\theta + h_2)\lambda(s)}{T\theta^2}[e^{\theta T} - \theta T - 1]
+ \frac{sI_e\lambda(s)}{2T} [2MT - (1 - \alpha)N^2 - T^2]
+ \frac{(h_2 - h_1)\lambda(s)}{\theta^2 T}[(\theta T - \theta t_1 + 1)e^{\theta t_1} - (\theta T + 1)]
\]
(10)

and

\[
TP_8(s, T) = (s - c) \lambda(s) - \frac{A}{T} - \frac{(c\theta + h_2)\lambda(s)}{T\theta^2}[e^{\theta T} - \theta T - 1] + \frac{sI_e\lambda(s)}{2T} [M^2 - (1 - \alpha)N^2]
+ \frac{(h_2 - h_1)\lambda(s)}{\theta^2 T}[(\theta T - \theta t_1 + 1)e^{\theta t_1} - (\theta T + 1)]
- \frac{cI_e\lambda(s)}{\theta^2 T}[e^{\theta(T - M)} - \theta(T - M) - 1].
\]
(11)

(B2) When \( N \leq t_1 \leq M, \)

\[
TP(s, T) = \begin{cases} 
TP_5(s, T) & \text{if } 0 < T < N, \\
TP_9(s, T) & \text{if } N < T \leq t_1, \\
TP_7(s, T) & \text{if } t_1 < T \leq M, \\
TP_8(s, T) & \text{if } M < T, \end{cases}
\]
(12a)

(12b)

(12c)

(12d)

where

\[
TP_9(s, T) = (s - c) \lambda(s) - \frac{A}{T} - \frac{(c\theta + h_2)\lambda(s)}{T\theta^2}[e^{\theta T} - \theta T - 1]
+ \frac{sI_e\lambda(s)}{2T} [2MT - (1 - \alpha)N^2 - T^2].
\]
(13)

(B3) When \( M < t_1, \)

\[
TP(s, T) = \begin{cases} 
TP_5(s, T) & \text{if } 0 < T \leq N, \\
TP_9(s, T) & \text{if } N < T \leq M, \\
TP_{10}(s, T) & \text{if } M < T \leq t_1, \\
TP_8(s, T) & \text{if } t_1 < T, \end{cases}
\]
(14a)

(14b)

(14c)

(14d)
where
\[
TP_{10}(s, T) = (s - c)\lambda(s) - \frac{A}{T} - \frac{(c\theta + h_1)\lambda(s)}{T^2} \left[ e^{\theta T} - \theta T - 1 \right] + \frac{c I_k \lambda(s)}{\theta^2 T} \left[ e^{\theta(T-M)} - \theta(T-M) - 1 \right] + \frac{s l e \lambda(s)}{2 T} [M^2 - (1 - \alpha) N^2]. 
\] (15)

3. The concavity of TP\(_1(s, T)\)

Equations (2), (3), (4), (6), (8), (9), (10), (11), (13) and (15) yield
\[
\frac{\partial TP_1(s, T)}{\partial T} = \frac{A}{T^2} - \frac{\lambda(s)(h_1 + c\theta)}{\theta^2 T^2} \left[ \theta T e^{\theta T} - e^{\theta T} + 1 \right] - \frac{s l e \alpha \lambda(s)}{2}, 
\] (16)

\[
\frac{\partial^2 TP_1(s, T)}{\partial T^2} = -2A T^{-3} - \frac{\lambda(s)(h_1 + c\theta)}{\theta^2 T^3} [\theta T e^{\theta T} - 2\theta T e^{\theta T} + 2 e^{\theta T} - 2] < 0, 
\] (17)

\[
\frac{\partial TP_2(s, T)}{\partial T} = \frac{A}{T^2} - \frac{\lambda(s)(h_2 + c\theta)}{\theta^2 T^2} \left[ \theta T e^{\theta T} - e^{\theta T} + 1 \right] + \frac{\lambda(s)(h_2 - h_1)}{\theta^2 T^2} \left[ \theta T_1 e^{\theta T_1} - e^{\theta T_1} + 1 \right] - \frac{s l e \alpha \lambda(s)}{2}, 
\] (18)

\[
\frac{\partial^2 TP_2(s, T)}{\partial T^2} = -2A T^{-3} - \frac{\lambda(s)(h_2 + c\theta)}{\theta^2 T^3} [\theta T e^{\theta T} - 2\theta T e^{\theta T} + 2 e^{\theta T} - 2] - \frac{2\lambda(s)(h_2 - h_1)}{\theta^2 T^3} [\theta T_1 e^{\theta T_1} - e^{\theta T_1} + 1] < 0, 
\] (19)

\[
\frac{\partial TP_3(s, T)}{\partial T} = \frac{A}{T^2} - \frac{\lambda(s)(h_2 + c\theta)}{\theta^2 T^2} \left[ \theta T e^{\theta T} - e^{\theta T} + 1 \right] + \frac{\lambda(s)(h_2 - h_1)}{\theta^2 T^2} \left[ \theta T_1 e^{\theta T_1} - e^{\theta T_1} + 1 \right] - \frac{s l e \alpha \lambda(s) M^2}{2 T^3}, 
\] (20)

\[
\frac{\partial^2 TP_3(s, T)}{\partial T^2} = -2A T^{-3} - \frac{\lambda(s)(h_2 + c\theta)}{\theta^2 T^3} [\theta T e^{\theta(T-M)} - e^{\theta(T-M)} - \theta M + 1] + \frac{2\lambda(s)(h_2 - h_1)}{\theta^2 T^3} \left[ \theta T_1 e^{\theta T_1} - e^{\theta T_1} + 1 \right] - \frac{c I_k \lambda(s)}{\theta^2 T^3} \left[ (\theta T)^2 e^{\theta(T-M)} - 2\theta T e^{\theta(T-M)} + 2 e^{\theta(T-M)} - 2 \right] + \frac{s l e \alpha \lambda(s) M^2}{T^3}, 
\] (21)
\[
\frac{\partial T P_4(s, T)}{\partial T} = A \frac{1}{T^2} - \frac{\lambda(s)(h_1 + c\theta)}{\theta^2 T^3} \left[ \theta T \, e^{\theta T} - e^{\theta T} + 1 \right] \\
- \frac{c I_k \lambda(s)}{\theta^2 T^2} \left[ (\theta T) e^{\theta(T-M)} - e^{\theta(T-M)} - \theta M + 1 \right] - \frac{s I_e \alpha \lambda(s) M^2}{2T^2},
\]

(22)

\[
\frac{\partial^2 T P_4(s, T)}{\partial T^2} = -2A \frac{1}{T^3} - \frac{\lambda(s)(h_1 + c\theta)}{\theta^2 T^3} \left[ (\theta T)^2 e^{\theta T} - 2\theta T \, e^{\theta T} + 2 e^{\theta T} - 2 \right] \\
+ \frac{c I_k \lambda(s)}{\theta^2 T^3} \left[ (\theta T)^2 e^{\theta(T-M)} - 2\theta T \, e^{\theta(T-M)} + 2 e^{\theta(T-M)} - 2 \right] \\
+ \frac{s I_e \alpha \lambda(s) M^2}{2T^3},
\]

(23)

\[
\frac{\partial T P_5(s, T)}{\partial T} = A \frac{1}{T^2} - \frac{\lambda(s)(h_1 + c\theta)}{\theta^2 T^3} \left[ \theta T \, e^{\theta T} - e^{\theta T} + 1 \right] - \frac{s I_e \alpha \lambda(s)}{2},
\]

(24)

\[
\frac{\partial^2 T P_5(s, T)}{\partial T^2} = -2A \frac{1}{T^3} - \frac{\lambda(s)(h_1 + c\theta)}{\theta^2 T^3} \left[ (\theta T)^2 e^{\theta T} - 2\theta T \, e^{\theta T} + 2 e^{\theta T} - 2 \right] < 0,
\]

(25)

\[
\frac{\partial T P_6(s, T)}{\partial T} = A \frac{1}{T^2} - \frac{\lambda(s)(h_2 + c\theta)}{\theta^2 T^2} \left[ \theta T \, e^{\theta T} - e^{\theta T} + 1 \right] + \frac{\lambda(s)(h_2 - h_1)}{\theta^2 T^2} \left[ \theta t_1 \, e^{\theta t_1} - e^{\theta t_1} + 1 \right] \\
- \frac{s I_e \alpha \lambda(s)}{2} e^{\theta t_1} - e^{\theta t_1} + 1 \right] - \frac{s I_e \alpha \lambda(s) N^2(1 - \alpha)}{2T^2},
\]

(26)

\[
\frac{\partial^2 T P_6(s, T)}{\partial T^2} = -2A \frac{1}{T^3} - \frac{\lambda(s)(h_2 + c\theta)}{\theta^2 T^3} \left[ (\theta T)^2 e^{\theta T} - 2\theta T \, e^{\theta T} + 2 e^{\theta T} - 2 \right] \\
+ \frac{2 \lambda(s)(h_2 - h_1)}{\theta^2 T^3} \left[ \theta t_1 \, e^{\theta t_1} - e^{\theta t_1} + 1 \right] < 0,
\]

(27)

\[
\frac{\partial T P_7(s, T)}{\partial T} = A \frac{1}{T^2} - \frac{\lambda(s)(h_2 + c\theta)}{\theta^2 T^2} \left[ \theta T \, e^{\theta T} - e^{\theta T} + 1 \right] \\
+ \frac{\lambda(s)(h_2 - h_1)}{\theta^2 T^2} \left[ \theta t_1 \, e^{\theta t_1} - e^{\theta t_1} + 1 \right] - \frac{s I_e \lambda(s) N^2(1 - \alpha)}{2T^2},
\]

(28)

\[
\frac{\partial^2 T P_7(s, T)}{\partial T^2} = -2A \frac{1}{T^3} - \frac{\lambda(s)(h_2 + c\theta)}{\theta^2 T^3} \left[ (\theta T)^2 e^{\theta T} - 2\theta T \, e^{\theta T} + 2 e^{\theta T} - 2 \right] \\
- \frac{2 \lambda(s)(h_2 - h_1)}{\theta^2 T^3} \left[ \theta t_1 \, e^{\theta t_1} - e^{\theta t_1} + 1 \right] - \frac{\lambda(s)s I_e N^2(1 - \alpha)}{T^3} < 0,
\]

(29)
\[
\frac{\partial\text{TP}_8(s, T)}{\partial T} = \frac{A}{T^2} - \frac{\lambda(s)(h_2 + c\theta)}{\theta^2 T^2} \left[ \theta T e^{\theta T} - e^{\theta T} + 1 \right] + \frac{\lambda(s)(h_2 - h_1)}{\theta^2 T^2} \left[ \theta t_1 e^{\theta t_1} - e^{\theta t_1} + 1 \right] - \frac{c I_k \lambda(s)}{\theta^2 T^2} \left[ \theta T e^{\theta(T-M)} - e^{\theta(T-M)} - \theta M + 1 \right] - \frac{s I_e \lambda(s)}{2T^2} \left[ M^2 - (1 - \alpha)N^2 \right],
\]

\[
\frac{\partial^2\text{TP}_8(s, T)}{\partial T^2} = -\frac{2A}{T^3} - \frac{\lambda(s)(h_2 + c\theta)}{\theta^2 T^3} \left[ (\theta T)^2 e^{\theta T} - 2\theta T e^{\theta T} + 2 e^{\theta T} - 2 \right] - \frac{c I_k \lambda(s)}{\theta^2 T^3} \left[ (\theta T)^2 e^{\theta(T-M)} - 2\theta T e^{\theta(T-M)} + 2 e^{\theta(T-M)} + 2\theta M - 2 \right] + \frac{s I_e \lambda(s)}{T^3} \left[ M^2 - (1 - \alpha)N^2 \right],
\]

\[
\frac{\partial\text{TP}_9(s, T)}{\partial T} = \frac{A}{T^2} - \frac{\lambda(s)(h_1 + c\theta)}{\theta^2 T^2} \left[ \theta T e^{\theta T} - e^{\theta T} + 1 \right] - \frac{s I_e \lambda(s)}{2T^2} + \frac{s I_e \lambda(s)N^2(1-\alpha)}{2T^2}.
\]

\[
\frac{\partial^2\text{TP}_9(s, T)}{\partial T^2} = -\frac{2A}{T^3} - \frac{\lambda(s)(h_1 + c\theta)}{\theta^2 T^3} \left[ (\theta T)^2 e^{\theta T} - 2\theta T e^{\theta T} + 2 e^{\theta T} - 2 \right] - \frac{s I_e \lambda(s)}{T^3} \left[ M^2 - (1 - \alpha)N^2 \right] < 0,
\]

\[
\frac{\partial\text{TP}_{10}(s, T)}{\partial T} = \frac{A}{T^2} - \frac{\lambda(s)(h_1 + c\theta)}{\theta^2 T^2} \left[ \theta T e^{\theta T} - e^{\theta T} + 1 \right] - \frac{c I_k \lambda(s)}{\theta^2 T^2} \left[ \theta T e^{\theta(T-M)} - e^{\theta(T-M)} - \theta M + 1 \right] - \frac{s I_e \lambda(s)}{2T^2} \left[ M^2 - (1 - \alpha)N^2 \right],
\]

\[
\frac{\partial^2\text{TP}_{10}(s, T)}{\partial T^2} = -\frac{2A}{T^3} - \frac{\lambda(s)(h_1 + c\theta)}{\theta^2 T^3} \left[ (\theta T)^2 e^{\theta T} - 2\theta T e^{\theta T} + 2 e^{\theta T} - 2 \right] - \frac{c I_k \lambda(s)}{\theta^2 T^3} \left[ (\theta T)^2 e^{\theta(T-M)} - 2\theta T e^{\theta(T-M)} + 2 e^{\theta(T-M)} + 2\theta M - 2 \right] + \frac{s I_e \lambda(s)}{T^3} \left[ M^2 - (1 - \alpha)N^2 \right].
\]

Let \( T_{i}^*(s) [i = 1, 2, 5, 6, 7, 9] \) denote the optimal cycle time of \( \text{TP}_i(s, T) \) on \( T > 0 \) and let \( T_{i}^*(s) [i = 3, 4, 8, 10] \) denote the optimal cycle time of \( \text{TP}_i(s, T) \) on \( T \geq M \) for any fixed \( s \). According to Thangam & Uthayakumar (2010) (16–35), we have the following results.

**Lemma 3.1**

(A) \( \text{TP}_i(s, T) \) is concave on \( T > 0 \) for any fixed \( s \) and \( i = 1, 2, 5, 6, 7, 9 \).

(B) If \( \frac{\partial\text{TP}_i(s, T)}{\partial T} > 0 \), then there exists a unique point \( T_{i}^*(s) \in (M, \infty) \) such that \( \text{TP}_i(s, T) \) is increasing on \( [M, T_{i}^*(s)] \) and decreasing on \( [T_{i}^*(s), \infty) \). Furthermore, if \( \frac{\partial\text{TP}_i(s, T)}{\partial T} \leq 0 \), then \( \text{TP}_i(s, T) \) is decreasing on \( [M, \infty) \) for any fixed \( s \) and \( i = 3, 4, 8, 10 \).
4. Theorem for the optimal cycle time $T^*(s)$ of $TP(s, T)$ when $s$ is fixed

Equations (16), (18), (20), (22), (24), (26), (28), (30), (32) and (34) yield that

$$\frac{\partial TP_1(s, t_1)}{\partial T} = \frac{\partial TP_2(s, t_1)}{\partial T} = \frac{\partial TP_3(s, t_1)}{\partial T} = \frac{\partial TP_4(s, t_1)}{\partial T} = \frac{\partial TP_5(s, t_1)}{\partial T} = \frac{\partial TP_6(s, t_1)}{\partial T} = \frac{\partial TP_7(s, t_1)}{\partial T} = \frac{\partial TP_8(s, t_1)}{\partial T} = \frac{\partial TP_9(s, t_1)}{\partial T} = \frac{\partial TP_{10}(s, t_1)}{\partial T},$$

where

$$\Delta_{12}(s) = 2\theta^2 A - 2\lambda(s)(h_1 + c\theta)[\theta t_1 e^{\theta t_1} - e^{\theta t_1} + 1] - s I e^{\alpha \lambda(s)}(\theta t_1)^2,$$

$$\Delta_{23}(s) = 2\theta^2 A - 2\lambda(s)(h_2 + c\theta)[\theta M e^{\theta M} - e^{\theta M} + 1] + 2\lambda(s)(h_2 - h_1)[\theta t_1 e^{\theta t_1} - e^{\theta t_1} + 1]$$

$$- s I e^{\alpha \lambda(s)}(\theta M)^2,$$
\[
\Delta_{14}(s) = 2\theta^2 A - 2\lambda(s)(h_1 + c\theta)\left[\theta M e^{\theta M} - e^{\theta M} + 1 \right] - s I_e a \lambda(s)(\theta M)^2, \\
\Delta_{43}(s) = 2\theta^2 A - 2\lambda(s)(h_1 + c\theta)\left[\theta t_1 e^{\theta t_1} - e^{\theta t_1} + 1 \right] \\
- 2c I_k \lambda(s) \left[\theta t_1 e^{\theta(t_1-M)} - e^{\theta(t_1-M)} - \theta M + 1 \right] - s I_e a \lambda(s)(\theta M)^2, \\
\Delta_{56}(s) = \Delta_{12}(s), \\
\Delta_{67}(s) = 2\theta^2 A - 2\lambda(s)(h_2 + c\theta)\left(\theta N e^{\theta N} - e^{\theta N} + 1 \right) \\
+ 2\lambda(s)(h_2 - h_1)\left(\theta t_1 e^{\theta t_1} - e^{\theta t_1} + 1 \right) - s I_e a \lambda(s)(\theta N)^2, \\
\Delta_{78}(s) = 2\theta^2 A - 2\lambda(s)(h_1 + c\theta)\left(\theta M e^{\theta M} - e^{\theta M} + 1 \right) \\
+ 2\lambda(s)(h_2 - h_1)\left(\theta t_1 e^{\theta t_1} - e^{\theta t_1} + 1 \right) - s I_e a \lambda(s)(\theta M)^2, \\
\Delta_{59}(s) = 2\theta^2 A - 2\lambda(s)(h_1 + c\theta)\left(\theta N e^{\theta N} - e^{\theta N} + 1 \right) - s I_e a \lambda(s)(\theta N)^2, \\
\Delta_{97}(s) = 2\theta^2 A - 2\lambda(s)(h_1 + c\theta)\left[\theta M e^{\theta M} - e^{\theta M} + 1 \right] - s I_e a \lambda(s)\theta^2 \left[\theta^2 - (1 - \alpha)N^2 \right] \\
\Delta_{910}(s) = 2\theta^2 A - 2\lambda(s)(h_1 + c\theta)\left[\theta M e^{\theta M} - e^{\theta M} + 1 \right] - s I_e a \lambda(s)\theta^2 \left[\theta^2 - (1 - \alpha)N^2 \right] \\
= 2\theta^2 A - 2\lambda(s)(h_1 + c\theta)\left[\theta M e^{\theta M} - e^{\theta M} + 1 \right] - s I_e a \lambda(s)\theta^2 \left[\theta^2 - (1 - \alpha)N^2 \right]
\]

Equations (47–57) imply that for any fixed \(s\),

(A) if \(M < N\) and \(t_1 \leq M\), then \(\Delta_{12}(s) \geq \Delta_{23}(s)\). 

(B) if \(M < N\) and \(t_1 > M\), then \(\Delta_{14}(s) > \Delta_{43}(s)\). 

(C) if \(M \geq N\) and \(t_1 < N\), then \(\Delta_{56}(s) > \Delta_{67}(s) > \Delta_{78}(s)\). 

(D) if \(M \geq N\) and \(N \leq t_1 \leq M\), then \(\Delta_{59}(s) > \Delta_{97}(s) > \Delta_{78}(s)\), 

(E) if \(M \geq N\) and \(M < t_1\), then \(\Delta_{59}(s) > \Delta_{910}(s) > \Delta_{108}(s)\).

Based on the above arguments, we have the following results.
THEOREM 4.1 For any fixed $s$, there are five situations to be explored.

(A) Suppose that $M < N$ and $t_1 \leq M$. Hence,

(A1) If $\Delta_{12}(s) < 0$, then $T^*(s) = T^*_1(s)$.

(A2) If $\Delta_{23}(s) < 0 \leq \Delta_{12}(s)$, then $T^*(s) = T^*_2(s)$.

(A3) If $\Delta_{23}(s) \geq 0$, then $T^*(s) = T^*_3(s)$.

(B) Suppose that $M < N$ and $t_1 > M$. Hence,

(B1) If $\Delta_{14}(s) < 0$, then $T^*(s) = T^*_1(s)$.

(B2) If $\Delta_{43}(s) < 0 \leq \Delta_{14}(s)$, then $T^*(s) = T^*_4(s)$.

(B3) If $\Delta_{43}(s) \geq 0$, then $T^*(s) = T^*_3(s)$.

(C) Suppose that $M \geq N$ and $t_1 < N$. Hence,

(C1) If $\Delta_{56}(s) < 0$, then $T^*(s) = T^*_5(s)$.

(C2) If $\Delta_{67}(s) < 0 \leq \Delta_{56}(s)$, then $T^*(s) = T^*_6(s)$.

(C3) If $\Delta_{78}(s) < 0 \leq \Delta_{67}(s)$, then $T^*(s) = T^*_7(s)$.

(C4) If $\Delta_{78}(s) \geq 0$, then $T^*(s) = T^*_8(s)$.

(D) Suppose that $M \geq N$ and $N \leq t_1 < M$. Hence,

(D1) If $\Delta_{59}(s) < 0$, then $T^*(s) = T^*_5(s)$.

(D2) If $\Delta_{97}(s) < 0 \leq \Delta_{59}(s)$, then $T^*(s) = T^*_9(s)$.

(D3) If $\Delta_{78}(s) < 0 \leq \Delta_{97}(s)$, then $T^*(s) = T^*_7(s)$.

(D4) If $\Delta_{78}(s) \geq 0$, then $T^*(s) = T^*_8(s)$.

(E) Suppose that $M \geq N$ and $M < t_1$. Hence,

(E1) If $\Delta_{59}(s) < 0$, then $T^*(s) = T^*_5(s)$.

(E2) If $\Delta_{910}(s) < 0 \leq \Delta_{59}(s)$, then $T^*(s) = T^*_9(s)$.

(E3) If $\Delta_{108}(s) < 0 \leq \Delta_{910}(s)$, then $T^*(s) = T^*_10(s)$.

(E4) If $\Delta_{108}(s) \geq 0$, then $T^*(s) = T^*_8(s)$.

Proof. See Appendix. \hfill \Box

5. The algorithm for locating the optimal solution $(s^*, T^*)$ of $\text{TP}(s, T)$

Referring to Chung (1994), we can develop the algorithm to locate the optimal solution $(s^*, T^*)$ of $\text{TP}(s, T)$ as follows.

The algorithm

Step 1: Input values of all parameters involved in the inventory model.

Step 2: Set $s = 0.01$, $s_{\text{stop}} = 0$, $s^*_{\text{opt}} = s$, $T^*_\text{opt} = T^*(s^*_{\text{opt}})$, $T^*_{\text{opt}} = \text{TP}(s^*_{\text{opt}}, T^*_\text{opt})$.

Step 3: If $M < N$, go to Step 4. Otherwise, go to Step 7.

Step 4: If $t_1 \leq M$, go to Step 5. Otherwise, go to Step 6.

Step 5: (1) If $\Delta_{12}(s) < 0$, set $T^*(s) = T^*_1(s)$ and go to Step 12.

(2) If $\Delta_{23}(s) < 0 \leq \Delta_{12}(s)$, set $T^*(s) = T^*_2(s)$ and go to Step 12.

(3) If $\Delta_{23}(s) \geq 0$, set $T^*(s) = T^*_3(s)$ and go to Step 12.
Step 6:  
(1) If \( \Delta_{14}(s) < 0 \), set \( T^*(s) = T_1^*(s) \) and go to Step 12.
(2) If \( \Delta_{43}(s) < 0 \leq \Delta_{14}(s) \), set \( T^*(s) = T_4^*(s) \) and go to Step 12.
(3) If \( \Delta_{13}(s) \geq 0 \), set \( T^*(s) = T_3^*(s) \) and go to Step 12.

Step 7:  
If \( t_1 \leq N \), go to Step 8. Otherwise, go to Step 9.

Step 8:  
(1) If \( \Delta_{56}(s) < 0 \), set \( T^*(s) = T_2^*(s) \) and go to Step 12.
(2) If \( \Delta_{67}(s) < 0 \leq \Delta_{56}(s) \), set \( T^*(s) = T_6^*(s) \) and go to Step 12.
(3) If \( \Delta_{78}(s) < 0 \leq \Delta_{67}(s) \), set \( T^*(s) = T_7^*(s) \) and go to Step 12.
(4) If \( \Delta_{78}(s) \geq 0 \), set \( T^*(s) = T_8^*(s) \) and go to Step 12.

Step 9:  
If \( N \leq t_1 \leq M \), go to Step 10. Otherwise, go to Step 11.

Step 10:  
(1) If \( \Delta_{59}(s) < 0 \), set \( T^*(s) = T_5^*(s) \) and go to Step 12.
(2) If \( \Delta_{97}(s) < 0 \leq \Delta_{59}(s) \), set \( T^*(s) = T_9^*(s) \) and go to Step 12.
(3) If \( \Delta_{78}(s) < 0 \leq \Delta_{97}(s) \), set \( T^*(s) = T_7^*(s) \) and go to Step 12.
(4) If \( \Delta_{78}(s) \geq 0 \), set \( T^*(s) = T_8^*(s) \) and go to Step 12.

Step 11:  
(1) If \( \Delta_{59}(s) < 0 \), set \( T^*(s) = T_5^*(s) \) and go to Step 12.
(2) If \( \Delta_{910}(s) < 0 \leq \Delta_{59}(s) \), set \( T^*(s) = T_9^*(s) \) and go to Step 12.
(3) If \( \Delta_{108}(s) < 0 \leq \Delta_{910}(s) \), set \( T^*(s) = T_{10}^*(s) \) and go to Step 12.
(4) If \( \Delta_{108}(s) \geq 0 \), set \( T^*(s) = T_{10}^*(s) \) and go to Step 12.

Step 12:  
(A) If \( TP_{opt}^{*} < TP(s, T^*(s)) \), then set \( s_{opt}^* = s, T_{opt}^* = T^*(s) \), \( TP_{opt}^* = TP(s_{opt}^*, T_{opt}^*) \), \( s_{stop} = 0, s = s + 0.01 \) and go to Step 3.
(B) If \( TP_{opt}^* \geq TP(s, T^*(s)) \). If \( s_{stop} > 30 \), go to Step 13. Otherwise, set \( s_{stop} = s_{stop} + 1, s = s + 0.01 \) and go to Step 3.

Step 13: Exit the optimal solution and the optimal value and set \( s^* = s_{opt}^*, T^* = T_{opt}^* \) and \( TP^* = TP_{opt}^*(s^*, T^*) \).

**Remark 5.1** For any fixed \( s \), all \( T_i^*(s) \) is obtained by executing the intermediate value theorem (varberg et al., 2007, pp. 87–88) on \( \partial TP_i(s, T) / \partial T \) for finding the unique solution such that \( \partial TP_i(s, T_i^*(s)) / \partial T = 0 \) if \( T_i^*(s) \) exists.

**6. Comparisons of sensitivity analyses between the present paper and Thangam & Uthayakumar (2010)**

Assume that the demand is a linearly decreasing function of selling price, i.e. \( \lambda(s) = 500 - 1.21s \). Furthermore, given \( Z = 150, A = 1240, \theta = 0.05, \alpha = 0.7, h_1 = 1, h_2 = 3, c = 100, I_k = 0.15, I_r = 0.12, N = 0.60 \) and \( M = 0.94 \). In order to evaluate the accuracy of the algorithm described in Section 5, the same sensitivity analyses as those in Thangam & Uthayakumar (2010) are executed. We obtain Table 1 to make comparisons between this paper and Thangam & Uthayakumar (2010).

From Table 1, we have the following observations:

(1) The optimal solution profit in this paper is better than that obtained by Thangam & Uthayakumar (2010).
(2) The optimal price, cycle time and ordering quantity decrease with parameter \( \alpha \). The optimal profit increases with parameter \( \alpha \).
TABLE 1 Comparisons between this paper and Thangam & Uthayakumar (2010)

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$TP^*_{TU}$ = the optimal profit of Thangam & Uthayakumar (2010); IMRP = \( \frac{TP^* - TP^*_{TU}}{TP^*_{TU}} \times 100\% \) = the improvement rate of profit.
(3) When $h_2 = 5$, the IMRP is 0.245%; when $h_2 = 8$, the IMRP is 0.665%; when $h_2 = 11$, the IMRP is 0.693%. Consequently, the profit improvement rate is significant. That is, the larger $h_2$ is, the larger IMRP is.

(4) The optimal price, cycle time and ordering quantity are not significantly affected by the value of $N$. The optimal profit decreases with parameter $N$.

(5) The optimal price and cycle time decrease with parameter $M$. The ordering quantity and optimal profit increase with parameter $M$.

(6) The optimal price, cycle time, ordering quantity and optimal profit all increase with parameter $Z$. Moreover, when $Z = 250$, the IMRP is 1.237%; when $Z = 350$, the IMRP is 0.806%; when $Z = 450$, the IMRP is 0.52%. Consequently, the profit improvement rate is significant with parameter $Z$.

(7) The optimal price increases with parameter $\theta$. The optimal cycle time, ordering quantity and profit decrease with parameter $\theta$. In addition, when $\theta = 0.25$, the IMRP is 0.044%; when $\theta = 0.35$, the IMRP is 0.125%. That is, the profit improvement rate is sensitive to charge in $\theta$.

(8) The optimal price, cycle time and ordering quantity increase with parameter $A$. The optimal profit decreases with parameter $A$.

(9) The optimal price and cycle time increase with parameter $c$. The optimal ordering quantity and profit decrease with parameter $c$.

(10) Executing times of all the above examples on a personal computer do not exceed 0.05 s. Basically, the algorithm described in Section 5 is rather rapid.

7. Managerial implications

The EOQ model is probably the oldest inventory control model. Osteryoung et al. (1986) reported that these models are still widely used in industry, although the assumptions (constant demand rate, known inventory holding and set-up cost, no shortages allowed, unit production cost independent of the lot size, constant production rate, infinite horizon) necessary to justify their use are rarely met. Obviously, the traditional EOQ model does not take the deterioration rate $\theta$ into account. However, in practice, if $\theta$ is small, deterioration can be ignored. Applying the traditional EOQ model to the inventory problem is enough. However, when concerned with deteriorating items, it is not appropriate to assume that $\theta$ is small. Thangam & Uthayakumar (2010) considered deteriorating items in their inventory models but assumed that $\theta T$ is small in order to determine an optimum policy. Our view is that if one is to consider deterioration, then it is better not to assume that $\theta T$ is small, and the solution procedure we develop therefore has a practical advantage.

8. Conclusions

Thangam & Uthayakumar (2010) investigated the two-warehouse inventory system for perishable items under partial trade credit policy. They aimed to simplify the solution procedure. Their results are based on the assumption that the number $\theta T$ should be small. Our paper derives an exact algorithm that provides a more accurate and reliable solution.

In fact, Huang (2005) discusses another kind of the partial trade credit, different from Huang & Hsu (2008), to allow that the supplier would offer the retailer partial credit but not full trade credit. In addition, Teng & Goyal (2007) presented another kind of two-level trade credit policy, different from

Acknowledgements

The authors are greatly indebted to Editors Philip Scarf and Aris A. Syntetos for providing valuable comments and suggestions to improve this article.

REFERENCES


**Appendix**

For any fixed $s$, there are five situations to be explored.

(A1) Suppose that $M < N$ and $t_1 \leq M$. Hence,

(a11) $TP_1(s, T)$ is increasing on $(0, T_1^*(s))$ and decreasing on $[T_1^*(s), t_1]$.

(a12) $TP_2(s, T)$ is decreasing on $[t_1, M]$.

(a13) $TP_3(s, T)$ is decreasing on $[M, \infty)$.

Combining 1(a, b, c) and (a11)–(a13), we conclude that $TP(s, T)$ is increasing on $(0, T_1^*(s))$ and decreasing on $[T_1^*(s), \infty)$. Therefore, $T^*(s) = T_1^*(s)$.

(A2) If $\Delta_{23}(s) < 0 \leq \Delta_{12}(s)$, then Lemma 3.1 implies that

(a21) $TP_1(s, T)$ is increasing on $(0, t_1)$.

(a22) $TP_2(s, T)$ is increasing on $[t_1, T_2^*(s)]$ and decreasing on $[T_2^*(s), M]$.

(a23) $TP_3(s, T)$ is decreasing on $[M, \infty)$.

Combining 1(a, b, c) and (a21)–(a23), we conclude that $TP(s, T)$ is increasing on $(0, T_2^*(s))$ and decreasing on $[T_2^*(s), \infty)$. Therefore, $T^*(s) = T_2^*(s)$.

(A3) If $\Delta_{23}(s) \geq 0$, then $\Delta_{12}(s) > \Delta_{23}(s) \geq 0$. So, Lemma 3.1 implies that

(a31) $TP_1(s, T)$ is increasing on $(0, t_1)$.
(a32) $\text{TP}_2(s, T)$ is increasing on $[t_1, M]$.

(a33) $\text{TP}_3(s, T)$ is increasing on $[M, T^*_3(s)]$ and decreasing on $[T^*_3(s), \infty)$.

Combining 1(a, b, c) and (a31)–(a33), we conclude that $\text{TP}(s, T)$ is increasing on $(0, T^*_3(s))$ and decreasing on $[T^*_3(s), \infty)$. Therefore, $T^*(s) = T^*_3(s)$. Based on the above arguments about (A1)–(A3), this completes the proof of Theorem 4.1(A).

(B) Suppose that $M < N$ and $t_1 > M$. With Lemma 3.1 and 5(a, b, c), adopting the similar techniques of proof as those in Theorem 4.1(A), we can demonstrate that Theorem 4.1(B) holds.

(C) Suppose that $M < N$ and $t_1 < N$. With Lemma 3.1 and 7(a, b, c, d), adopting the similar techniques of proof as those in Theorem 4.1(A), we can demonstrate that Theorem 4.1(C) holds.

(D) Suppose that $M \geq N$ and $N \leq t_1 \leq M$. With Lemma 3.1 and 12(a, b, c, d), adopting the similar techniques of proof as those in Theorem 4.1(A), we can demonstrate that Theorem 4.1(D) holds.

(E) Suppose that $M \geq N$ and $t_1 > M$. With Lemma 3.1 and 14(a, b, c, d), adopting the similar techniques of proof as those in Theorem 4.1(A), we can demonstrate that Theorem 4.1(E) holds.

Incorporating all discussions about (A)–(E), we have completed the proof of Theorem 4.1.